



DESIGN AND IMPLEMENTATION OF A MAGNETIC LEVITATION SYSTEM CONTROLLER USING GLOBAL SLIDING MODE CONTROL

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Abstract

This paper presents global sliding mode control and conventional sliding mode control for stabilization position of a levitation object. Sliding mode control will be robusting when in sliding mode condition. However, it is not necessarily robust at attaining phase. In the global sliding mode control, the attaining motion phase was eliminated, so that the robustness of the controller can be improved. However, the value of the parameter uncertainties needs to be limited. Besides that, the common problem in sliding mode control is high chattering phenomenon. If the chattering is too large, it can make the system unstable due the limited ability of electronics component. The strategy to overcome the chattering phenomenon is needed. Based on simulation and experimental results, the global sliding mode control has better performance than conventional sliding mode control.

Keywords: magnetic levitation system, global sliding mode control, conventional sliding mode control, chattering.

I. INTRODUCTION

Magnetic levitation systems (MLS) have been widely applied in various fields that aims to help people, such as biomedical [1], maglev train [2], maglev wind tunnel [3], and micro-robotic [4]. Several techniques have been implemented to control the magnetic levitation system, for examples using gain scheduling [5], high-gain observers [6], and passivity based control [7, 8]. These controllers do not consider the parameter uncertainties of magnetic levitation system. However, some researchers have used nonlinear controllers considering mass parameter uncertainties of magnetic levitation such as using gain scheduling [5], and sliding mode control [9]. The uncertainty of parameters other than mass is not considered. In this paper, not only uncertainty of the mass but also uncertainties of the other parameters are considered.

Linear controller is not suitable for control magnetic levitation because the dynamic of magnetic levitation is highly nonlinear. Nonlinear controller is needed to overcome the nonlinearity

of magnetic levitation. Besides that, general problem in MLS is uncertainties of the system. The uncertainties are a very challenging task for researchers. These problems can be overcome with robust controller. Robust controller are composed of a nominal part, similar to a feedback linearization or inverse control law, and of additional terms aimed at dealing with model uncertainty [10].

Sliding mode control technique will be investigated to control position object of MLS. Sliding mode control is precise controller to solve the problems of the system, because this technique are nonlinear controller and robust from disturbances and parameter uncertainties. Sliding mode control technique was initially proposed in the early 1960s. The ideas did not appear outside Russia until mid 1970s [11].

The MLS consists of an object suspended in a voltage controlled magnetic field. The magnetic field is used to against gravity effect. To adjust the desired position of the object, we can use a controller to change appropriate magnetic field. Model of MLS is shown in Figure 1.

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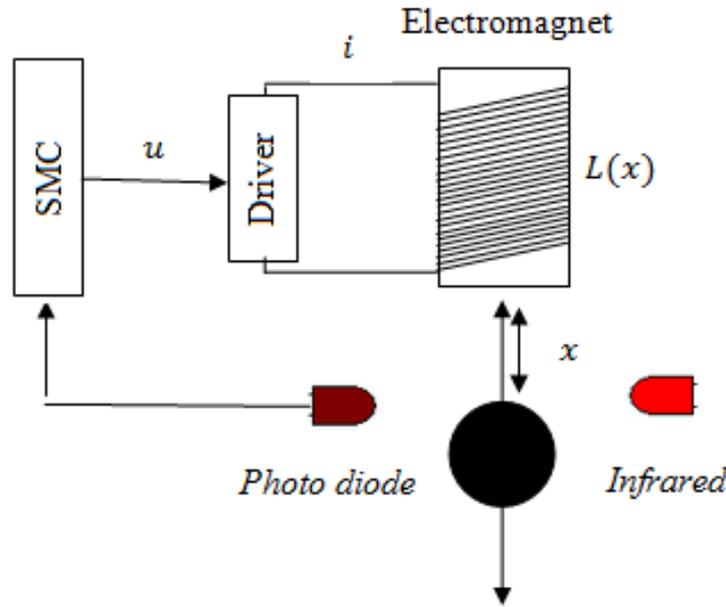


Figure 1. Model of MLS

II. DYNAMICS OF THE MLS

The Lagrangian will be presented to find the mathematical model of a MLS. Lagrange equations of motion can be written as:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{x}_i} \right) - \frac{\partial L}{\partial x_i} = 0, 1 \leq i \leq n. \quad (1)$$

We now define $L = T - V$; L is called the Lagrangian, T is kinetic energy and V is potential energy.

The generalized coordinates to be chosen such that $x_1 = x$, $\dot{x}_1 = \dot{x}$, $\dot{x}_2 = i$. x represent the object position, \dot{x} represent the velocity, and i represent the current. The kinetic energy and potential energy can be written as:

$$\left. \begin{aligned} T &= \frac{1}{2} L(x) i^2 + \frac{1}{2} m \dot{x}^2, \\ V &= -mgx, \end{aligned} \right\} \quad (2)$$

where coil inductivity $L(x)$ is a nonlinear function of the ball's position, g is the gravitational constant, and m is the mass of a levitated object.

The Lagrangian formula can be written as:

$$L = \frac{1}{2} L(x) i^2 + \frac{1}{2} m \dot{x}^2 + mgx. \quad (3)$$

The approximation coil inductance $L(x)$ can be written as:

$$L(x) = L_i + \frac{L_0 x_0}{x}, \quad (4)$$

where $L_0 x_0 = 2k$, L_i is a system parameter, and k is the magnetic force constant.

Thus, the general form of Lagrangian equation can be written as:

$$\ddot{x} = \frac{f}{m} + g - k \frac{i^2}{m x^2}, \text{ and} \quad (5)$$

$$\frac{di}{dt} = \frac{u}{L} - \frac{iR}{L} + i \frac{2k}{L x^2} \dot{x}. \quad (6)$$

Taking $x_1 = x$, $x_2 = \dot{x}$, $x_3 = i$, $u = e$. Then, we can get state space model of a MLS as:

$$\left. \begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= g - \frac{k x_3^2}{m x_1^2}, \\ \dot{x}_3 &= -\frac{R x_3}{L} + \frac{2k x_2 x_3}{L x_1^2} + \frac{u}{L}. \end{aligned} \right\} \quad (7)$$

Consider the nonlinear change of coordinates. The nonlinear change in coordinates is presented in Equation (8).

$$\left. \begin{aligned} n_1 &= x_1 - x_{1d}, \\ n_2 &= x_2 - x_{2d}, \\ n_3 &= g - \frac{k x_3^2}{m x_1^2}, \text{ where } x_1 > 0 \text{ and } x_3 > 0. \end{aligned} \right\} \quad (8)$$

In the new coordinates, we can get:

$$\left. \begin{aligned} \dot{n}_1 &= n_2, \\ \dot{n}_2 &= n_3, \\ \dot{n}_3 &= f(n) + g(n)u. \end{aligned} \right\} \quad (9)$$

Differentiating Equation (9), then substituting Equation (7) into \dot{n}_3 :

$$\dot{n}_3 = -\frac{4k^2 x_2 x_3^2}{m L x_1^4} + \frac{2k R x_3^2}{m L x_1^2} + \frac{2k x_3^2 x_2}{m x_1^3} - \frac{2k x_3 u}{m x_1^2 L}. \quad (10)$$

The function $f(n)$ and $g(n)$ are correspondent to the original coordinates:

$$\left. \begin{aligned} f(x) &= -\frac{4k^2 x_2 x_3^2}{m L x_1^4} + \frac{2k R x_3^2}{m L x_1^2} + \frac{2k x_3^2 x_2}{m x_1^3}, \\ g(x) &= -\frac{2k x_3}{m x_1^2 L}, \text{ where } x_1 > 0. \end{aligned} \right\} \quad (11)$$

III. CONTROLLER DESIGN

Sliding mode control was first proposed by Emel'yanov and Barbhasin in the early 1960s [11]. The major advantages of this technique are completely insensitive to variation in system parameters, external disturbances, and modeling errors. The methodology of sliding mode control consists of three components. The first is designing a sliding surface in the state space. The second is designing high switching control law to reach the sliding surface. The third is designing equivalent control law to maintain the system state trajectory on the sliding surface for all subsequent times.

A. Conventional Sliding Mode Control

The first step in conventional sliding mode control (CSMC) is designing the switching surface. The sliding surface is defined as:

$$s = \ddot{n}_1 + \alpha \dot{n}_1 + \beta \tilde{n}_1, \quad (12)$$

substituting Equation (8) into Equation (12):

$$s = g - \frac{kx_3^2}{mx_1^2} + \alpha(x_2 - x_{2d}) + \beta(x_1 - x_{1d}). \quad (13)$$

The switching control law can be written as:

$$u_s = -A \operatorname{sign}(s), \quad (14)$$

where

$$\operatorname{sign}(s) = \begin{cases} 1, & s > 0, \\ 0, & s = 0, \\ -1, & s < 0. \end{cases} \quad (15)$$

The sliding mode control technique generally experiences chattering phenomenon, which is an oscillation around sliding surface as an effect of high switching. The saturation function can be used to overcome chattering phenomenon. The technique can be written as:

$$u_s = -A \operatorname{sat}(s), \quad (16)$$

where

$$\operatorname{sat}\left(\frac{s}{\varphi}\right) = \begin{cases} 1, & \frac{s}{\varphi} > 1, \\ \frac{s}{\varphi}, & \left|\frac{s}{\varphi}\right| \leq 1, \\ -1, & \frac{s}{\varphi} < -1. \end{cases} \quad (17)$$

The equivalent control design to maintain the system state trajectory is:

$$\dot{s} = \ddot{n}_1 + \alpha \dot{n}_1 + \beta \dot{n}_1. \quad (18)$$

substituting Equation (9) and (11) into Equation (18):

$$\dot{s} = f(x) + g(x)u_{eq} + \alpha\left(g - \frac{kx_3^2}{mx_1^2}\right) + \beta(x_2 - x_{2d}), \quad (19)$$

and we get equivalent control:

$$u_{eq} = \frac{1}{g(x)}(-f(x) - \alpha(g - \frac{kx_3^2}{mx_1^2}) - \beta(x_2 - x_{2d})). \quad (20)$$

Finally, we can get the CSMC controller as:

$$u = \frac{1}{g(x)}(-f(x) - \alpha(g - \frac{kx_3^2}{mx_1^2}) - \beta(x_2 - x_{2d})) - \operatorname{Asat}\left(\frac{s}{\varphi}\right). \quad (21)$$

The stability of a controller can be analyzed by Lyapunov function. Lyapunov function candidate is defined as:

$$V = \frac{1}{2}s^2 > 0, \quad (22)$$

and $\dot{V} = s\dot{s}$

$$\dot{V} = s(f(x) + g(x)u_{eq} + \alpha(g - \frac{kx_3^2}{mx_1^2}) + \beta(x_2 - x_{2d})), \quad (23)$$

Substituting Equation (21) into u_{eq} in Equation (23):

$$\begin{aligned} \dot{V} = s\dot{s} = s & \left(f(x) + g(x) \frac{1}{g(x)}(-f(x) - \alpha(g - \frac{kx_3^2}{mx_1^2}) - \beta(x_2 - x_{2d})) - \operatorname{Asat}\left(\frac{s}{\varphi}\right) \right. \\ & \left. + \alpha(g - \frac{kx_3^2}{mx_1^2}) + \beta(x_2 - x_{2d}) \right). \end{aligned} \quad (24)$$

It can be simplified as:

$$\begin{aligned} \dot{V} = s\dot{s} & < 0 \\ = s & \left(f(x) + g(x) \frac{1}{g(x)}(-f(x)) - \operatorname{Asat}\left(\frac{s}{\varphi}\right) \right). \end{aligned} \quad (25)$$

We ensure the stability of our system by choosing A to be large enough so that stable in the sense of Lyapunov.

B. Global Sliding Mode Control

The robustness of sliding mode control to disturbances and parameter uncertainties exists in sliding mode condition, and not necessarily robust at attaining phase. In the global sliding mode control (GSMC), the attaining motion phase was eliminated, so that the robustness of the controller can be improved [12-14].

The first step is designing the switching surface. The sliding surface is defined as:

$$s = \ddot{n}_1 + \alpha \dot{n}_1 + \beta n_1 - r(t), \quad (26)$$

substituting Equation (8) into Equation (26):

$$s = g - \frac{kx_3^2}{mx_1^2} + \alpha(x_2 - x_{2d}) + \beta(x_1 - x_{1d}) - r(t). \quad (27)$$

The additional function $r(t)$ should be satisfied:

$$r(0) = \ddot{e}_0 + \alpha \dot{e}_0 + \beta e_0, \quad (28a)$$

$$r(t) \rightarrow 0 \text{ as } t \rightarrow \infty, \quad (28b)$$

$$r(t) \in \mathbb{R}', \quad (28c)$$

where $e_0 = e(t = 0)$, $\alpha > 0$, and $\beta > 0$.

Equation (28a) represents the initial states on the sliding surface, Equation (28b) represents asymptotic stability, and Equation (28c) represents the existence of sliding mode (GSMC improved design for a brush). From three conditions to Equation (28a), (28b), and (28b), $r(t)$ can be designed as:

$$r(t) = r(0)e^{-kt}. \quad (29)$$

The switching control law can be written as:

$$u_s = -\left(\frac{1}{\Delta g(x)}(-\Delta f(x) - \alpha(g - \frac{kx_3^2}{mx_1^2}) - \beta(x_2 - x_{2d}) - \dot{r}) + D\right) \text{sat} \frac{s}{\varphi}. \quad (30)$$

The equivalent control to maintain the system state trajectory can be written as:

$$u_{eq} = \frac{1}{g(x)}(-\widehat{f(x)} - \alpha(g - \frac{kx_3^2}{mx_1^2}) - \beta(x_2 - x_{2d}) + \dot{r}), \quad (31)$$

where

$$\left. \begin{aligned} \widehat{R} &= \frac{R_{max} + R_{min}}{2}, \\ \Delta R &= \frac{R_{max} - R_{min}}{2}, \\ \widehat{L} &= \frac{L_{max} + L_{min}}{2}, \\ \Delta L &= \frac{L_{max} - L_{min}}{2}, \\ \widehat{m} &= \frac{m_{max} + m_{min}}{2}, \\ \Delta m &= \frac{m_{max} - m_{min}}{2}, \\ D &> 0, \\ \widehat{f(x)} &= -\frac{4k^2x_2x_3^2}{\widehat{m}Lx_1^4} + \frac{2k\widehat{R}x_3^2}{\widehat{m}Lx_1^2} + \frac{2kx_3^2x_2}{\widehat{m}x_1^3}, \\ \widehat{g(x)} &= -\frac{2kx_3}{\widehat{m}x_1^2L}, \\ f(x) &= -\frac{4k^2x_2x_3^2}{\Delta m\Delta Lx_1^4} + \frac{2k\Delta Rx_3^2}{\Delta m\Delta Lx_1^2} + \frac{2kx_3^2x_2}{\Delta mx_1^3}, \\ \Delta g(x) &= -\frac{2kx_3}{\Delta mx_1^2\Delta L}. \end{aligned} \right\} \quad (32)$$

Finally, we can get the GSMC controller:

$$u = \frac{1}{g(x)}(-\widehat{f(x)} - \alpha(g - \frac{kx_3^2}{mx_1^2}) - \beta(x_2 - x_{2d}) + \dot{r}) - \left(\frac{1}{\Delta g(x)}(-\Delta f(x) - \alpha(g - \frac{kx_3^2}{mx_1^2}) - \beta(x_2 - x_{2d}) - \dot{r}) + D\right) \text{sat} \frac{s}{\varphi}. \quad (33)$$

Lyapunov function candidate is defined as:

$$V = \frac{1}{2}s^2 > 0, \quad (34)$$

and

$$\dot{V} = s\dot{s} = s(f(x) + g(x)u_{eq} + \alpha(g - \frac{kx_3^2}{mx_1^2}) + \beta(x_2 - x_{2d}) - \dot{r}). \quad (35)$$

Substituting Equation (33) into u_{eq} in Equation (35):

$$\begin{aligned} \dot{V} &= s(f(x) + g(x)\frac{1}{g(x)}(-\widehat{f(x)} - \alpha(g - \frac{kx_3^2}{mx_1^2}) - \\ &\beta(x_2 - x_{2d}) + \dot{r}) - (\frac{1}{\Delta g(x)}(-\Delta f(x) - \\ &\alpha(g - \frac{kx_3^2}{mx_1^2}) - \beta(x_2 - x_{2d}) - \dot{r}) + D) \text{sat} \frac{s}{\varphi} + \\ &\alpha(g - \frac{kx_3^2}{mx_1^2}) + \beta(x_2 - x_{2d}) - \dot{r}). \end{aligned} \quad (36)$$

It can be simplified:

$$\dot{V} = s\dot{s} < 0 = s(f(x) + g(x)\frac{1}{g(x)}(-\widehat{f(x)} - (\frac{1}{\Delta g(x)}(-\Delta f(x) - \dot{r}) + D) \text{sat} \frac{s}{\varphi}). \quad (37)$$

We ensure the stability of our system choosing D to be large enough so that stable in the sense of Lyapunov.

IV. SIMULATION RESULTS

The robustness of the CSMC and GSMC from parameter uncertainties and disturbance are proposed. Furthermore, the chattering phenomenon will be investigated. The parameters of the magnetic levitation system are the gravitational $g = 9.81 \text{ m/s}^2$, the mass of the object $m = 340 \text{ g}$, the coil's resistance $R = 7.3 \Omega$, the magnetic force constant $13 \cdot 10^{-5} \text{ Nm}^2/\text{A}^2$, and the inductance $L = 0.089 \text{ H}$.

The simulation results of chattering phenomenon are shown in Figure 2 and Figure 3. The simulation results of CSMC are shown in Figure 2 to Figure 7. The simulation results of GSMC are shown in Figure 8 to Figure 11.

The chattering phenomenon is considering as a problem in sliding mode control. The simulation in Figure 2 shows the controller has high chattering phenomenon and Figure 3 shows the position of the object can follow the reference. Although the object can follow the reference, in actual plant, chattering phenomenon will make the system unstable due the limited ability of electronics component. The chattering phenomenon can be reduced by saturated function.

Figure 4 is control versus time for the system without uncertainties. Figure 5 shows the object can follow the reference very well. The matches between mathematical model of the controller and dynamics of the magnetic levitation make the system stable. Figure 6 and Figure 7 are simulation results of the system with uncertainties and disturbance. The parameters in Figure 6 and Figure 7 are the mass of the object $m = 340 \text{ g} + 20 \text{ g}$, the coil's resistance $R = 7.3 \Omega + 1.7 \Omega$, and the inductance $L = 0.089 \text{ H} + 0.032 \text{ H}$. The cosine disturbance is $-5 * \cos(5 * t) \text{ V}$.

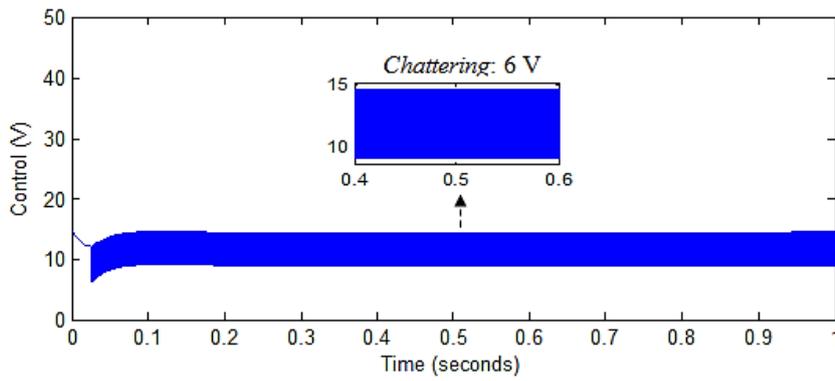


Figure 2. Control with signum function (CSMC)

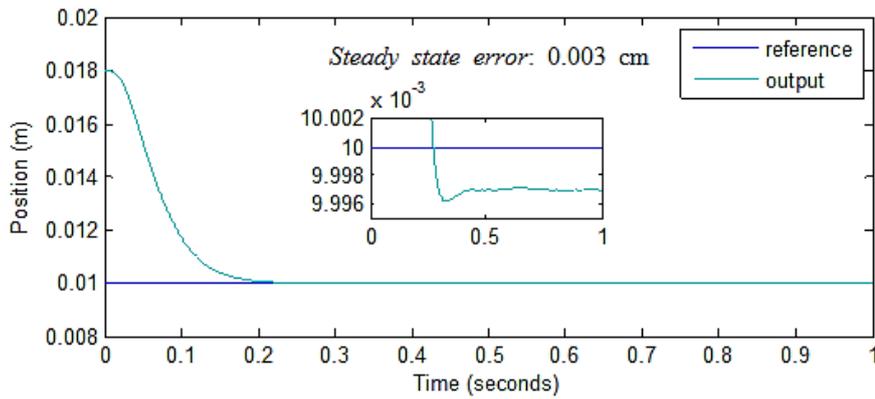


Figure 3. Output with signum function (CSMC)

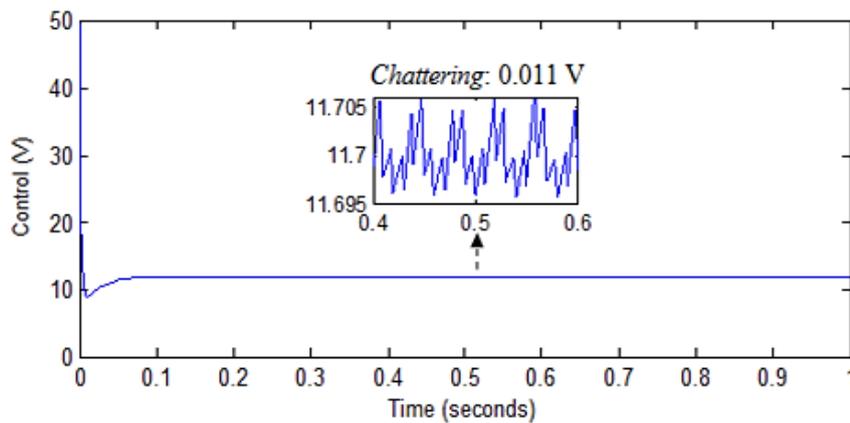


Figure 4. Control without parameter uncertainty and disturbance (CSMC)

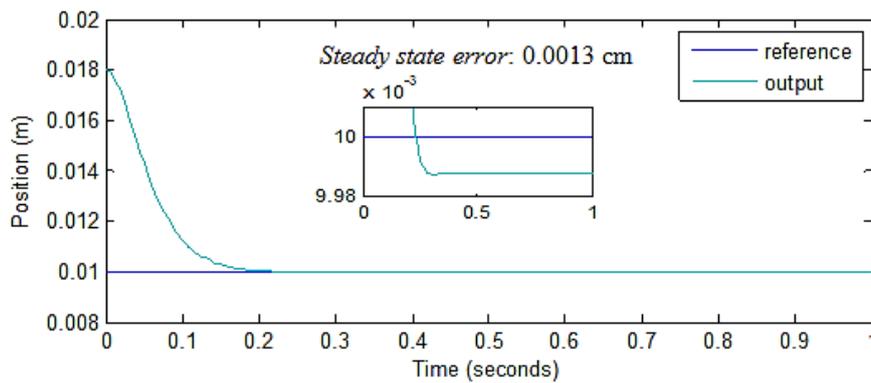


Figure 5. Output without parameter uncertainty and disturbance (CSMC)

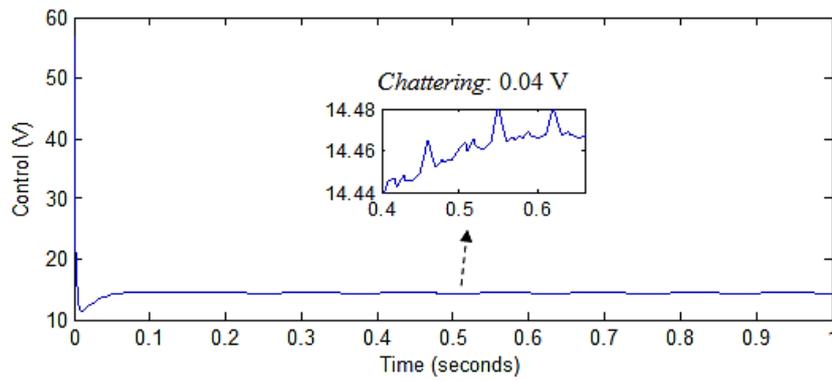


Figure 6. Control with parameter uncertainties and disturbance (CSMC)

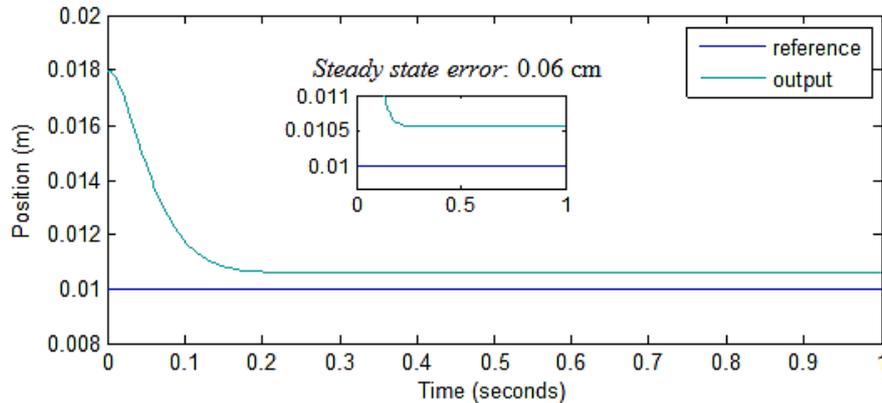


Figure 7. Output with parameter uncertainties and disturbance (CSMC)

Figure 7 shows that the object can follow the reference but have larger steady state error than the system without parameter uncertainties and disturbances.

Simulation results of GSMC are shown in Figure 8 to Figure 11. Figure 8 is control versus time for the system without uncertainties. The position object in Figure 9 shows that the object can follow the desired reference and have faster response than CSMC.

The robustness of controller from disturbance and parameter uncertainties are shown in Figure 10 and Figure 11. Figure 11 shows the object can follow the desired reference more closely than CSMC. The parameters in Figure 10 and Figure 11 are mass of the object $m = 340\text{ g} + 20\text{ g}$, the coil's resistance $R = 7.3\ \Omega + 1.7\ \Omega$, and the inductance $L = 0.089\text{ H} + 0.032\text{ H}$. Besides that the cosine disturbance $-5 * \cos(5 * t)\text{ V}$ is given in Figure 10 and Figure 11.

V. EXPERIMENTAL RESULTS

Matlab Simulink with block set embedded target for microchip device was used to construct program in the microcontroller. The experiment set up consists of iron ball, dsPIC33FJ128MC802 microcontroller with 16 bit resolution, electromagnet coil, and infrared-photodiode sensor. The output and control was recorded by DAQ 6009 with 1,000 Hz sample rate. The set

point position of the object is 1 cm. Parameters values were conditioned as follow. Nominal mass was $m = 340\text{ g}$, and it was supposed that $R = 7.3\ \Omega$ and $L = 0.089\text{ H}$. Mass deviation was $m = 340\text{ g} + 20\text{ g}$. Uncertainties of R and L occur naturally when the temperature of the coil increases. External disturbance was undefined. They can make the system unstable if the controller is not really robust.

Experimental results of CSMC are shown in Figure 12 to Figure 15. Figure 12 describes control versus time for the system without uncertainties. Figure 13 shows the object can follow the reference. The matches between mathematical model of the controller and dynamics of the magnetic levitation make the system stable. Figure 14 represents control versus time for the system with uncertainties. Figure 15 shows the object cannot follow the reference 1 cm because the CSMC is not really overcome the parameter uncertainties.

Experimental results of GSMC are shown in Figure 16 to Figure 19. Figure 16 is control versus time for the system without uncertainties. The position object in Figure 17 shows the object can follow the desired reference. Figure 18 is control versus time for the system with uncertainties. The position object in Figure 19 shows the object can follow the set point although the parameter uncertainties occur.

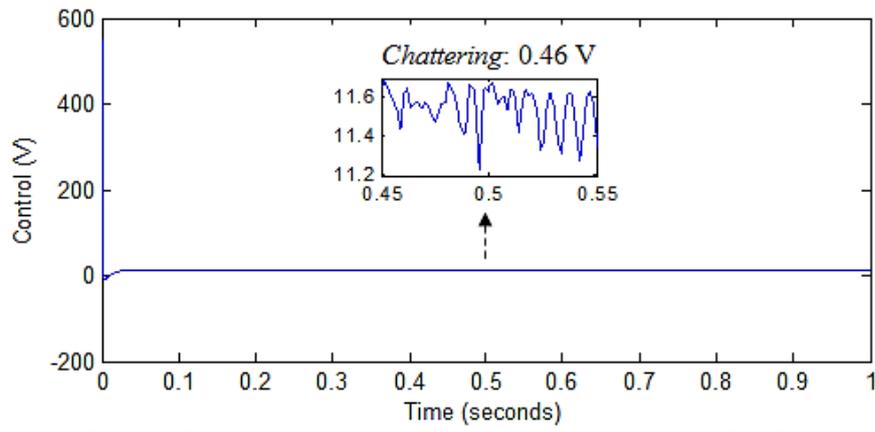


Figure 8. Control without parameter uncertainty and disturbance (GSMC)

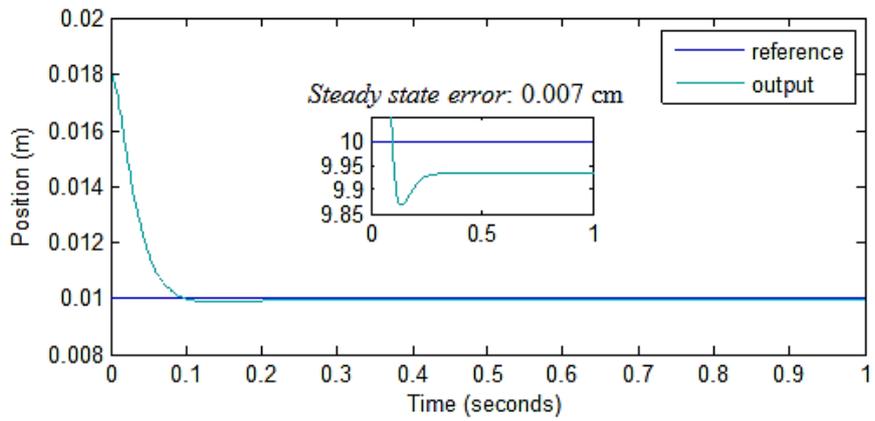


Figure 9. Output without parameter uncertainty and disturbance (GSMC)

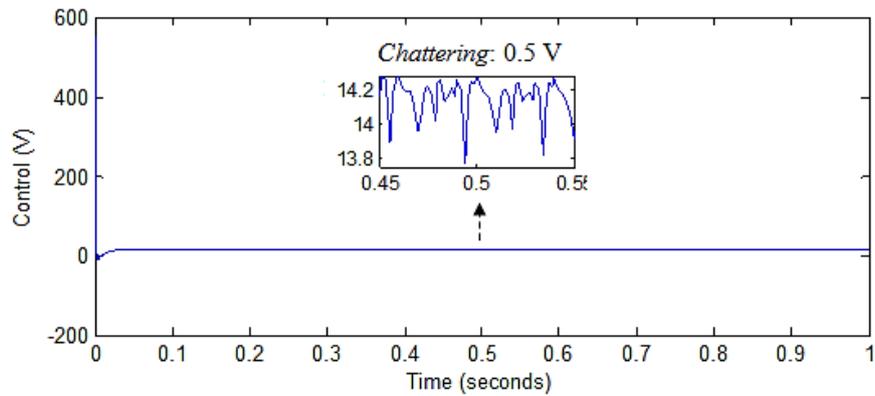


Figure 10. Control with parameter uncertainties and disturbance (GSMC)

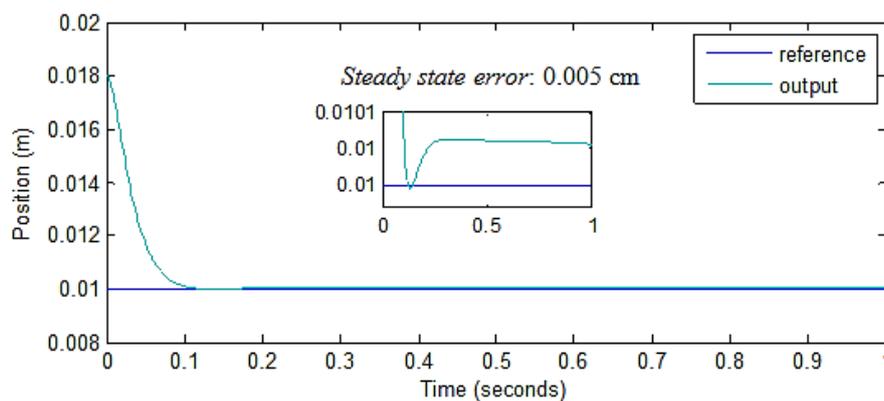


Figure 11. Output with parameter uncertainties and disturbance (GSMC)

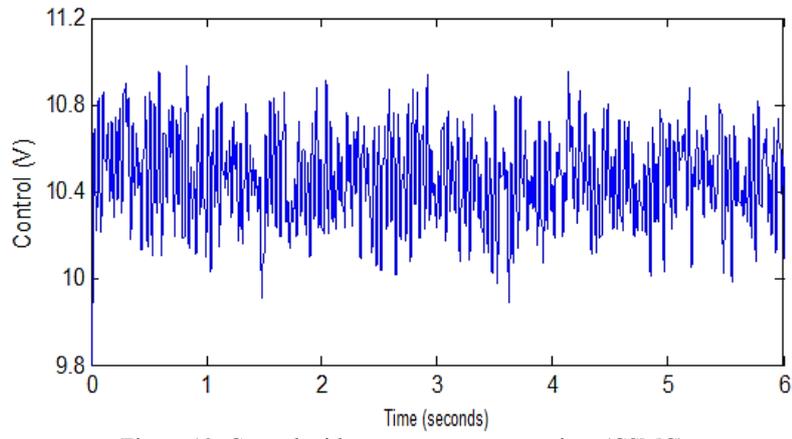


Figure 12. Control without parameter uncertainty (CSMC)

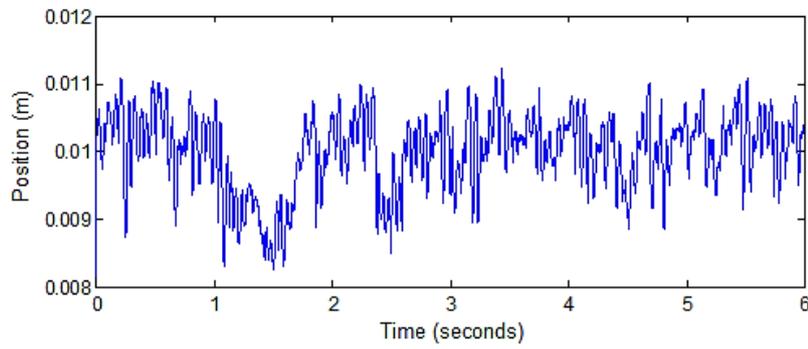


Figure 13. Output without parameter uncertainty (CSMC)

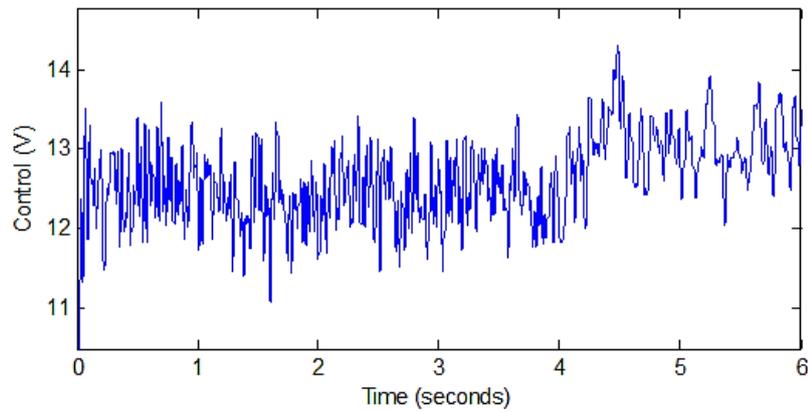


Figure 14. Control with parameter uncertainties (CSMC)

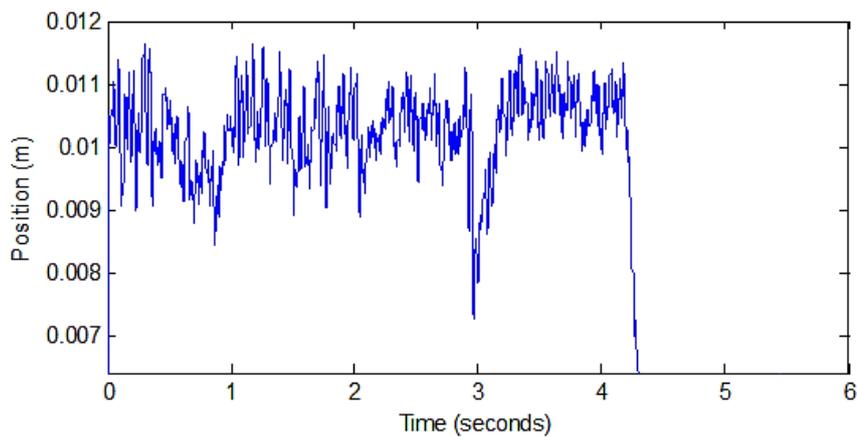


Figure 15. Output with parameter uncertainties (CSMC)

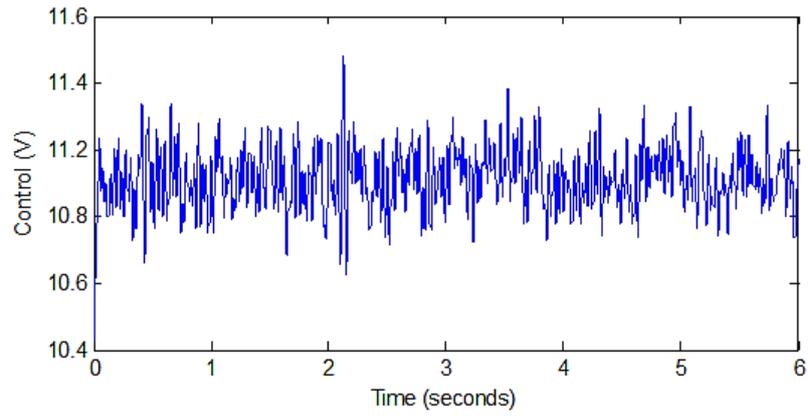


Figure 16. Control without parameter uncertainty (GSMC)

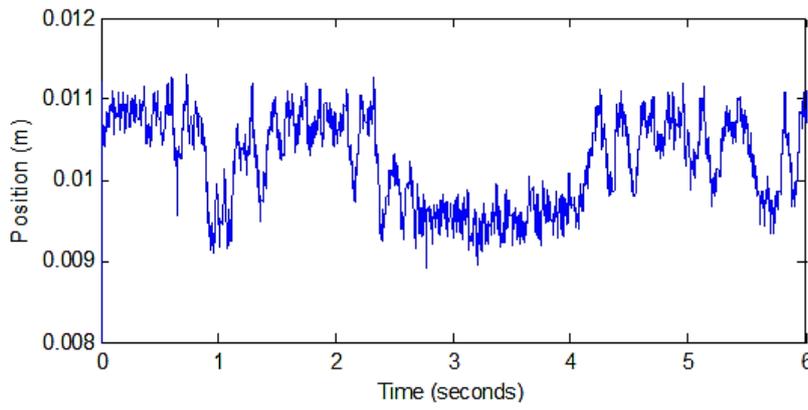


Figure 17. Output without parameter uncertainties (GSMC)

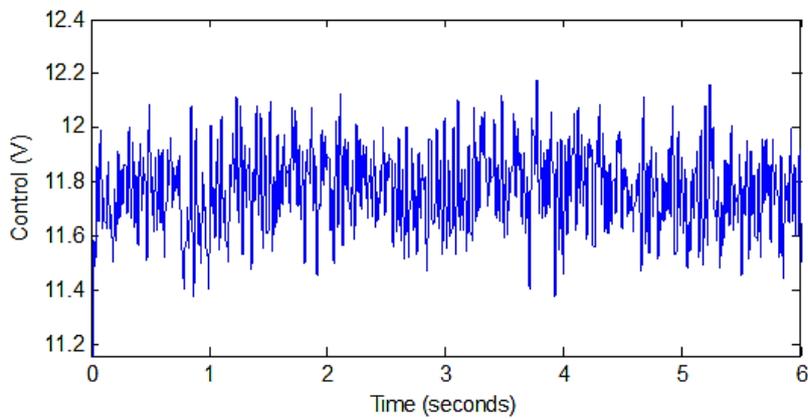


Figure 18. Control with parameter uncertainties (GSMC)

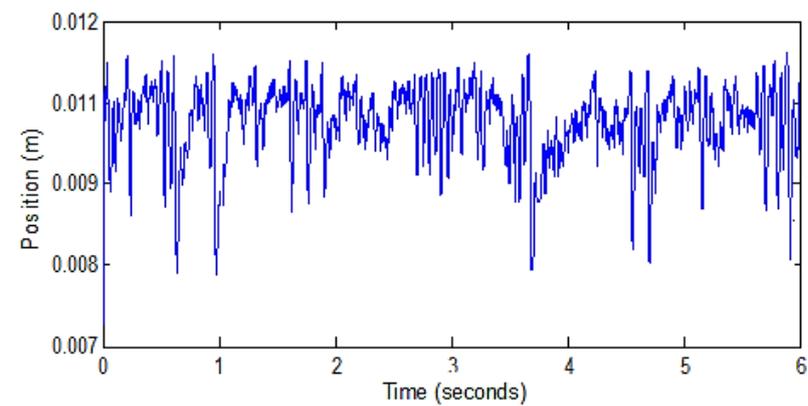


Figure 19. Output with parameter uncertainties (GSMC)

VI. CONCLUSION

This paper described the robustness of CSMC and GSMC from disturbances and parameter uncertainties. The CSMC is not necessarily robust at attaining phase. The mismatches between mathematical model of the controller and dynamics of the magnetic levitation make the system unstable. The GSMC shows good performance from disturbances and uncertainties. This technique can eliminate the mismatches between mathematical model of the controller and dynamics of the magnetic levitation. In the GSMC, the attaining motion phase was eliminated, so that the robustness of the controller can be improved. However, the value of the parameter uncertainties needs to be limited. Based on simulation and experimental results, the GSMC has better performance than CSMC.

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