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Stability analysis of a hybrid DC-DC buck converter model using dissipation inequality and convex optimization

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Abstract

The stability analysis of a DC-DC buck converter is a challenging problem due to the hybrid systems characteristic of its dynamics. Such a challenge arises from the buck converter operation which depends upon the ON/OFF logical transitions of its electronic switch component to correspondingly activate different continuous vector fields of the converter's temporal dynamics. This paper presents a sum of squares (SOS) polynomial optimization approach for stability analysis of a hybrid model of buck converter which explicitly takes into account the converter's electronic switching behavior. The proposed method first transforms the converter's hybrid dynamics model into an equivalent polynomial differential algebraic equation (DAE) model. An SOS programming algorithm is then proposed to computationally prove the stability of the obtained DAE model using Lyapunov's stability concept. Based on simulation results, it was found that the proposed method requires only 8.5 seconds for proving the stability of a buck converter model. In contrast, exhaustive simulations based on numerical integration scheme require 15.6 seconds to evaluate the stability of the same model. These results thus show the effectiveness of the proposed method as it can prove the converter stability in shorter computational times without requiring exhaustive simulations using numerical integration.

Keywords: DC-DC buck converter; switched hybrid systems; Lyapunov method; dissipation inequality; SOS programming.

I. Introduction

A DC-DC converter is an electronic device which transfers electric power from a DC voltage source to the loads [1]. Such a transfer is achieved through the activation/inactivation of an electronic switch which causes the electric power to be transmitted from the source to power storage devices when the switch is activated (ON) and then subsequently transferred from the storage device to the load when the switch is inactivated (OFF). The electronic switch is typically made of transistor and/or diode while the power storage devices usually consist of capacitor and/or inductor. The result of these power transfer processes is the converters output voltage whose value is proportional to the ratio of the durations of the ON and OFF states of the switch [2]. In practice, there are two types of converters that are used in electronic applications, namely the step down (or *buck*) and step up (*boost*) converters. For a given source voltage value, the buck converter produces a lower output voltage while the boost converter generates a higher one. In this paper, our focus is to study and analyze the dynamics of a buck converter due to its frequent and widespread uses in household and industrial electronic devices which range from simple motor control [3] to the design of photovoltaic power systems [4] and electric vehicles [5].

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Based on its working principle, the buck converter can be viewed and modeled as switched hybrid systems (SHS) whose dynamics may switch/jump from one discrete mode/state of operation into another in accordance to the ON/OFF mode or state of its switch [6][7]. In particular, during the activation of either ON or OFF mode, the converter state variables (e.g. current or voltage) evolve continuously in time according to the vector fields which define these states trajectories. The hybrid characteristics of a buck converter often give rise to nonlinear behaviors that are complex and at times difficult to characterize [8]. As a result, much of prior analysis works on buck converter dynamics were often done using their so-called averaged model for which the switching behaviors can simply be neglected [9]. While the use of this averaged model has so far resulted in various stability analysis and control synthesis methods, the fact that the construction of such a model essentially relies on the linearization/approximation methods limits their applicability to relatively small operational regions [10]. These suggest that more works remain needed to better understand the hybrid dynamics of buck converters [11][12].

This paper proposes the use of a computational method based on SOS programming techniques [13] for analyzing the stability of a hybrid buck converter model. In the proposed method, the converter dynamics are first modeled as a two-mode SHS in which the activation of each mode is triggered by the ON/OFF state of the switch. The stability of the obtained SHS model is analyzed using sufficient stability conditions in the form of a dissipation inequality [14]. An SOS program to find a Lyapunov function which satisfies the formulated dissipation inequality (thus certifies the SHS stability) is then formulated [15]. Numerical simulation results which illustrate the effectiveness of the proposed computational method are then presented.

II. Materials and Methods

A. System description and model

Consider the schematic of a DC-DC buck converter in Figure 1 [1]. In this figure, V_g is the voltage source whose value needs to be decreased to meet the desired output voltage value at the resistor load R. Both the inductor L and the capacitor C serve as temporary power storage elements for the input voltage from V_g before being subsequently transferred to the load R as an output voltage. An additional resistor r_L as shown in the schematic is added to describe parasitic electrical current/voltage which may occurs in the converter circuitry. The transfer of electrical power from the input V_g to the output R which occurs in two subsequent modes is controlled by the sequence of activation of the electronic switches S_1 and S_2 as discussed below.

In the first mode (denoted as mode 0), switch S_1 is activated (ON) while switch S_2 is deactivated (OFF). In this case, the voltage source (V_g), the storage elements (L and C) and the load (R) are connected and form two electrical loops. To model the dynamics in this mode, define a vector of state variables $x = [i_L, v_C]^T$ which consists of the current that passes through the inductor L and the voltage across the capacitor C. Using Kirchoff's laws, it can be shown that the dynamics of the converters state variables satisfy the following equation (1) [2],

$$\dot{x}(t) = A_0x(t) + B_0u(t) = \begin{bmatrix} -r_L/C & -1/L \\ 1/C & -1/RC \end{bmatrix} x(t) + \begin{bmatrix} 1/L \\ 0 \end{bmatrix} u(t) \quad (1)$$

where $\dot{x}(t) = \frac{dx(t)}{dt}$ denotes the time derivative of the state variables (no unit), A_0 and B_0 are constant state and input matrices (no units), r_L is the parasitic resistance (ohm) in the circuit, $u(t) = V_g$ has been defined as the system's input. In the second mode (denoted as mode 1), switch S_2 is activated (ON) whereas switch S_1 is deactivated (OFF). In this case, the voltage source (V_g) is disconnected from both the power storage components (L and C) and the load (R). This implies that the two loops in mode 0 no longer include V_g as their elements. As a result, the dynamics of the converter's state variables in mode 1 is simply governed by equation (2),

$$\dot{x}(t) = A_1x(t) + B_1u(t) = \begin{bmatrix} -r_L/C & -1/L \\ 1/C & -1/RC \end{bmatrix} x(t) \quad (2)$$

where $B_1 = [0,0]^T$ by the mode definition.

Based on the above two operational modes, the buck converter dynamics may be modeled as an SHS model of the form equation (3) [16],

$$x(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) \quad (3)$$

where $x(t) \in \Re^2$ is the vector of state variables, $A_{\sigma(t)}$ and $B_{\sigma(t)}$ are the values of the constant state and

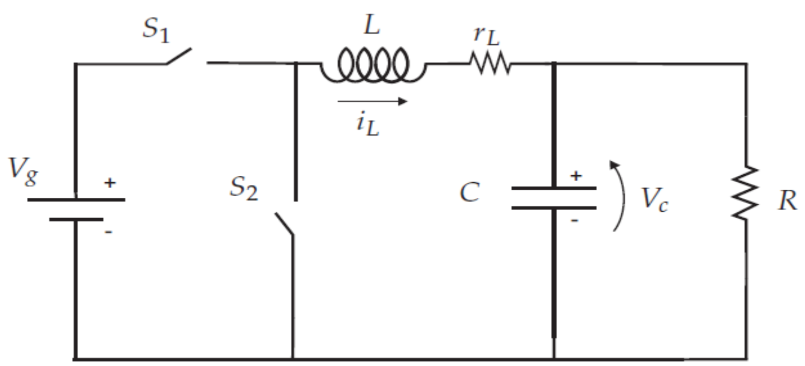

Figure 1. The schematic of a DC-DC buck converter

input matrices (no units) of the SHS at switching signal value $\sigma(t)$, u(t) is the (control) input and $\sigma(t): t \rightarrow \{0, 1\}$ is a switching signal which controls the mode that should be activated for a certain duration of time. It is thus clear that the SHS in equation (3) consists of two modes with similar state variables such that it reduces to equation (1) if $\sigma(t) = 0$ or simplifies to equation (2) when $\sigma(t) = 1$. In practice, the value of $\sigma(t)$ is usually regulated using a controller (e.g. pulse-width modulator) which sets the ratio of the time durations of the ON/OFF states of each switch in term of a duty ratio parameter [17].

The presence of the switching signal $\sigma(t)$ makes the analysis of the converter dynamics in equation (3) challenging. For instance, it is known that the overall dynamics of equation (3) can be unstable even if its subsystems are all stable. For this reason, considerable research efforts have been given in the last few decades to develop methods for analyzing the stability of SHS in equation (3) [18]. Currently, there are at least two main methods to do such analysis, i.e. using common Lyapunov function (CLF) [19] and multiple Lyapunov functions (MLF) [20] methods. Although theoretical basis for these methods have been established, their tractable computational implementations remain relatively unexplored. As in the case of standard Lyapunov-based methods, this lack of computational implementation has mainly been caused by the difficulty in finding the corresponding CLF or MLF [21]. This difficulty arises due to the fact that these methods essentially boil down to a problem of finding a nonnegative function that satisfies a set of nonlinear inequalities/equalities [22]. Finding such a function is known to be a computationally hard problem because there currently does not exist provable algorithms with polynomial time complexity to solve it [23].

To address the above difficulty, this paper proposes the use of SOS optimization techniques for analyzing the stability of the SHS in equation (3). The proposed method first transforms the hybrid dynamics of the buck converter into an equivalent polynomial differential algebraic equation (DAE) model [24]. Using the obtained DAE, this paper adopts a method from [14] to derive a dissipation inequality which defines the stability conditions of the resulting DAE form. Finally, an SOS programming approach [25] for computing a Lyapunov function which satisfies such an inequality is formulated.

B. DAE representation of switched hybrid systems

We next describe a method to construct an equivalent DAE model to represent the SHS model in equation (3). Let $\Re_+$ and $\Re^n$ denote the sets of nonnegative real numbers and n-dimensional Euclidean space, respectively. Consider a general SHS model in equation (4),

$$\dot{x}(t) = f_{\sigma(t)}(x(t), u(t)) \tag{4}$$

where $x(t) \in \Re^n$ and $u(t) \in \Re^m$ denote the state and control vectors of the SHS, respectively. $f_{\sigma(t)}$ is a nonlinear function describing the vector fields of the system (no unit) when the switching signal $\sigma(t)$ occurs. The function $\sigma(t): [0, t_f) \rightarrow \Theta \in \{0,1, \cdots, q\}$ is the switching signal which is a piecewise constant function of time, and $f_i(\cdot) \in \Re^n \times \Re^m \times \Re_+ \rightarrow \Re^n$ denotes a nonlinear function of the system vector fields when mode $i \in \Theta$ is active.

To define a DAE representation of equation (4), one first constructs a (row) drift vector $F(x, u)$ consisting the SHS's vector fields for all modes as in equation (5),

$$F(x, u) := [f_0(x, u) \quad f_1(x, u) \quad \cdots \quad f_q(x, u)] \tag{5}$$

Next, let $\Gamma(\sigma)$ be the quotient vector of the Lagrange polynomial interpolation of $F(x, u)$ in equation (5) of the form equation (6),

$$\Gamma(\sigma) := [\ell_0(\sigma) \quad \ell_1(\sigma) \quad \cdots \quad \ell_q(\sigma)] \tag{6}$$

where ℓ is the quotient Lagrange polynomial vector with elements ℓ_j where $j = 0, \dots, q$, in which each element of $L(\sigma)$ is defined in the switching variable σ as equation (7),

$$\ell_j(\sigma) = \prod_{\substack{i=0 \\ j=0}}^{q} \frac{(\sigma - i)}{(j - i)} \tag{7}$$

where ℓ_j for $j = 0, \dots, q$ is the element of the quotient Lagrange polynomial vector. The switching variable σ in the quotient vector equation (6) is constrained to take only integer values using polynomial function constraint in equation (8),

$$D(\sigma) := \prod_{j=0}^{q} (\sigma - j) = 0 \tag{8}$$

where $D(\sigma)$ is a polynomial function constraint for the quotient Lagrange polynomial of the switching function. In this regard, an equivalent representation of equation (3) in the form of polynomial DAE model can be constructed using $F(x, u)$, $\Gamma(\sigma)$, and $D(\sigma)$ in equation (5), equation (6), and equation (8), respectively as equation (9) [13],

$$\begin{aligned} \dot{x}(t) &= F(x, u)\Gamma(\sigma) \\ 0 &= D(\sigma) \end{aligned} \tag{9}$$

C. SOS programming

SOS programming is a variant of convex relaxation techniques in the context of polynomial optimization methods. The main idea in SOS programming methods is the reformulation of equality/inequality constraints in the considered problem as SOS polynomial conditions. Let Z_+ be the set of nonnegative integers and consider a polynomial ring $\Re[x]$ with unknown variables $x \in \Re^n$ and real-valued coefficients [26]. Recall that a polynomial function $V(x) \in \Re[x]$ being an SOS polynomial implies that $V(x)$ is also a positive definite (PD) function (i.e. $V(x) \geq 0$ for all $x \in \Re^n$). This implication in turn allows one to recast the determination of whether a polynomial function is SOS or not as semidefinite programming (SDP) problems. Specifically, a polynomial function $V(x)$ of degree 2d with $d \in Z_+$ is an SOS polynomial if there exist a PD matrix Q_s and a vector of monomials $\Psi(x)$ of degree $\leq d$ such that $V(x)$ can be decomposed as in equation (10) [27],

$$V(x) = \Psi^T(x) Q_s \Psi(x) \tag{10}$$

A key important point in equation (10) is that the construction of such a decomposition may be formulated and solved using SDP methods [28]. Specifically, by specifying the vector of finite degree monomials $\Psi(x)$, the construction of the decomposition in equation (10) boils down to the search for a positive definite matrix Q_s for which the equality in equation (10) holds [29]. This means various computational tools and solvers of semidefinite programming problems can be used to compute such a decomposition.

The decomposition in equation (10) forms the basis for the formulation of an SOS program. For instance, equation (10) can be used to simultaneously (i) determine if a polynomial $V(x)$ is PD and (ii) compute a positive lower bound $\gamma > 0$ for $V(x)$ using the SOS program in equation (11),

$$\begin{aligned} &min \quad \gamma \\ &\text{s.t.} \quad V(x) - \gamma \text{ is SOS} \end{aligned} \qquad (11)$$

Note that if the solution γ in equation (11) is feasible, then the SOS property of $V(x)$ guarantees that $V(x) - \gamma \geq 0$ holds, which thus implies $V(x)$ is a PD function that is lower bounded by the constant $\gamma > 0$. Particularly, equation (11) is a convex SDP problem as it searches for a constant $\gamma > 0$ and a PD matrix Q_s such that $V(x) - \gamma = \Psi^T(x) Q_s \Psi(x)$ holds. As such, various well-established computational tools in SDP methods can used to solve equation (11) [30].

III. Results and Discussions

A. DAE representation of buck converter model

For the SHS model in equation (2), the polynomial DAE representation in equation (9) can be constructed by noting that the system mode has such that. Thus, the drift vector in equation (5) for this case is defined as $F(x,u) := [A_0 x(t) \quad A_1 x(t)]$.

On the other hand, the elements of the quotient polynomial interpolation are defined as

$$\ell_0(\sigma) = \prod_{\substack{i=1 \\ j=0}}^{1} \frac{(\sigma-1)}{(0-1)} = 1 - \sigma, \qquad \ell_1(\sigma) = \prod_{\substack{i=0 \\ j=1}}^{1} \frac{(\sigma-0)}{(1-0)} = \sigma$$

such that the polynomial function is defined as $D(s) := \prod_{j=0}^{1} (\sigma - j) = \sigma(\sigma - 1)$.

As a result, an equivalent polynomial DAE representation of the SHS equation (2) is defined as equation (12),

$$\begin{aligned} \dot{x}(t) &= [A_0 x(t) + B_0 u(t)](1-\sigma) + A_1 x(t)\sigma \\ &= (A_0(1-\sigma) + A_1\sigma)x(t) + B_0(1-\sigma)u(t) \\ 0 &= \sigma(\sigma - 1) \end{aligned} \qquad (12)$$

B. Stability analysis of DAE system using dissipation inequality

This section describes an SOS programming formulation of a dissipation inequality which describes the sufficient stability conditions for the DAE representation in equation (10) of the SHS model in equation (3). In particular, this paper examines the use of Lyapunov's stability analysis method for studying the dynamics and stability of the DAE system in equation (10). To begin with, consider a general model of nonlinear systems in equation (13),

$$\dot{x}(t) = f(x(t)) + g(x(t))u(t), \qquad x(0) = x_0 \qquad (13)$$

The equilibrium $x^* \equiv \{x | f(x) + g(x)u = 0\}$ of equation (5) is said to be Lyapunov stable if there exists a function $V(x): \Re^n \to \Re_+$ which satisfies: (i) $V(x) \geq 0$ and (ii) $(\nabla_x V)[f(x) + g(x)u(t)] < 0$ for all $x \in \Re^n$ in which $(\nabla_x V) = (\partial V(x)/\partial x)$ [31]. For the DAE system of the form equation (10), Lyapunov stability analysis method may still be used through its reformulation in the form of a dissipation inequality. In this case, the system equilibrium vector $[x^*, \sigma]^T$ for a given u is defined as that for which conditions (i) $0 = [A_0 x^* + B_0 u(T)](1-\sigma) + A_1 x^*$ and (ii) $0 = \sigma(\sigma - 1)$ hold. The following theorem from [14] establishes a sufficient stability condition for system equation (12).

Theorem 1 [14]: The equilibrium $x^*$of the DAE in equation (4) is asymptotically stable if there exist a function $V(x): \Re^n \to \Re_+$, a scalar-valued function $\lambda(x,\sigma) > 0$ and a function $\Gamma(\sigma) = 0$ such that the following dissipation inequality holds around x^*,

$$(\nabla_x V)[A_0 x + B_0 u](1-\sigma) + A_1 x] < \lambda(x,\sigma)\Gamma^2(\sigma) \qquad (14)$$

Theorem 1 essentially states that if a set of functions $\{V(x), \lambda(x,\sigma), \Gamma(\sigma)\}$ which satisfy the inequality in equation (14) exist simultaneously, then the equilibrium of DAE in equation (12) is guaranteed to be asymptotically stable. Unfortunately, such a search is known to be a computationally hard problem. However, if $V(x)$, $\lambda(x,\sigma)$, and (σ) are polynomial functions, a tractable computation method for their search is available using techniques from SOS programming. Stability of SHS in equation (12) can be examined using SOS programming that corresponds to the result in Theorem 1.

C. SOS programming algorithm for stability of DAE model

Proposition 1 formulates an SOS program based on the result in Theorem 1. The main idea in this algorithm formulation is to relax the inequality constraints in equation (14) into SOS polynomial constraints.

Proposition 1: The equilibrium of the SHS model in equation (12) is asymptotically stable if there exist a polynomial function $V(x) \in \Re[x]$ *SOS*, polynomial functions $\lambda(\cdot) \in \Re[x,\sigma]$ and $\Gamma(\cdot) \in \Re[\sigma]$ such that the solutio$n\,\gamma > 0$ of the SOS program in equation (15) to equation (19) is feasible,

$$\min \quad \gamma$$

$$\text{s.t.} \quad V(x) - \gamma \quad \text{is SOS} \qquad (15)$$

$$\Gamma^2(\sigma)\lambda(x,\sigma) - (\nabla_x V)\dot{x}(t) \quad \text{is SOS} \qquad (16)$$

$$\lambda(x,\sigma) \quad \text{is SOS} \qquad (17)$$

$$\Gamma(\sigma) \quad \text{is SOS} \qquad (18)$$

$$-\Gamma(\sigma) \quad \text{is SOS} \qquad (19)$$

Proof: Assume the solution of equation (9) is feasible. Then there exists a constant $\gamma > 0$ which satisfies equation (15) to equation (19). Such a satisfaction thus particularly implies the existence of functions $\lambda(x,\sigma) \geq 0$ and $\Gamma(\sigma) = 0$. Moreover, the satisfaction of equation (15) implies the existence of a PD function $V(x) \geq \gamma > 0$ with a lower bound of $\gamma > 0$. Finally, the satisfaction of equation (16) implies $\lambda(x,\sigma)\Gamma^2(\sigma) - (\nabla_x V)[A_0 x + B_0 u](1-\sigma) + A_1 x] \geq 0$, which is essentially the condition in equation (14). By Theorem 1, we conclude that the equilibrium of the SHS equation (12) is asymptotically stable. The proof is thus completed.

Algorithm 1 details a computational method for the implementation of Proposition 1. Notice in this algorithm that $\Gamma(\sigma) = \sigma(\sigma - 1)$ is explicitly defined even though it may also be defined as an unknown polynomial function in variable σ that needs to be searched simultaneously with $V(x)$, γ and $\lambda(x,\sigma)$ during the optimization's iteration. This simply means that the decision variables of the optimization become larger. The explicit choice of $\Gamma(\sigma) = \sigma(\sigma - 1)$ in Algorithm 1 may thus be viewed as a way to reduce the computational load which otherwise may increase very fast when $\Gamma(\sigma)$ is left as decision variable. Algorithm 1 can be implemented in SOS programming tools in conjunction with SDP solvers [32]. Section III.D illustrates an implementation of Algorithm 1 for the SHS model in equation (4).

D. Simulation experiments

This section reports the simulation results of the implementation of Algorithm 1 to analyze the stability of the SHS model in equation (4). In the simulation, the model parameters of $V_g = 12$ volt, $R = 50k\,\Omega$, $r_L = 20.25\,\Omega$, $L = 0.33mH$ and $C = 120\,\mu F$ were used. The SHS model is assumed to operate with a duty cycle of 0.5. Algorithm 1 is implemented in MATLAB [33] programming platform using SOSTOOL [27] and MOSEK [34] software tools under a Core-i7, 4.2 GHz PC with 16 GB RAM. For the SHS model equation (4), Algorithm 1 was solved in 8.5 seconds and gives a Lyapunov function $V(x)$ of degree $d_V = 6$ and an SOS function $\lambda(x,\sigma)$ of degree $d_\lambda = 6$. The existence of such functions thus certify the asymptotic stability of the SHS model in equation (10). For comparison, simulation experiments were also conducted for the dynamics of the SHS model in equation (1) and equation (12) using a direct numerical integration method, as well as the buck converter dynamics based on the physical circuit in Figure 1.

The switching input signal is generated using a pulse width modulator (PWM) generator with a frequency of $f_{PWM} = 2KHz$. The measurable output of the system is assumed to be the output voltage across R. The simulations were conducted using MATLAB which is already integrated with SIMULINK and SIMSCAPE. Figure 2 shows the block diagram of the three buck converter models that are used in the simulation.

For the assumed model parameter values, simulations result of these models were obtained in 15.6 seconds which is longer than that required by the SOS programming method. As shown in Figure 3, the simulation results indicate similar stable behavior of the output voltages and reflect a resulting output voltage of 6 volts for the 12 volt input with 0.5 duty cycle. This thus verifies the stability property of the considered buck converter system as concluded by the existence of solution to the SOS Programming method in Algorithm 1. The main advantage of using SOS Programming method is that it essentially mimics the feature of Lyapunov's stability analysis method whereby the stability of a system can be inferred/concluded based on the existence of Lyapunov function and without having to rely on exhaustive simulation based on numerical integration methods.

Algorithm 1.
SOS program formulation in Proposition 1

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SOS Program for Stability Analysis of SHS Model in equation (4)
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Input : Matrices A_0, A_1 and B_1 for the SHS model in equation (10)

Output: Polynomial functions $V(x)$, $\lambda(\bar{x})$ and a lower bound γ

Initialization:

1. Define the polynomial $\Gamma(\sigma) = \sigma(1\text{-}\sigma)$
2. Set u to be a constant duty ratio input for the buck converter model

SOS Program

3. Define the vector of augmented decision variables $\bar{x} = [x, \sigma]^T$
4. Construct a polynomial function template $V(x) = \Sigma_{\alpha \leq d_V} c_\alpha x^\alpha$ of degree d_V with unknown coefficients c_α in which α is a multi-index
5. Construct a polynomial function template $\lambda(\bar{x}) = \Sigma_{\beta \leq d_\lambda} c_\beta \bar{x}^\beta$ of degree d_λ with unknown coefficients c_β in which β is a multi-index
6. Define a positive constant γ as the decision variable of the problem in equation (8)
7. Declare the SOS constraints (9b)-(9f) for the defined $V(x)$, $\lambda(\bar{x})$ and $\Gamma(\sigma)$
8. Solve the SOS program to find $V(x)$, $\lambda(\bar{x})$ and γ

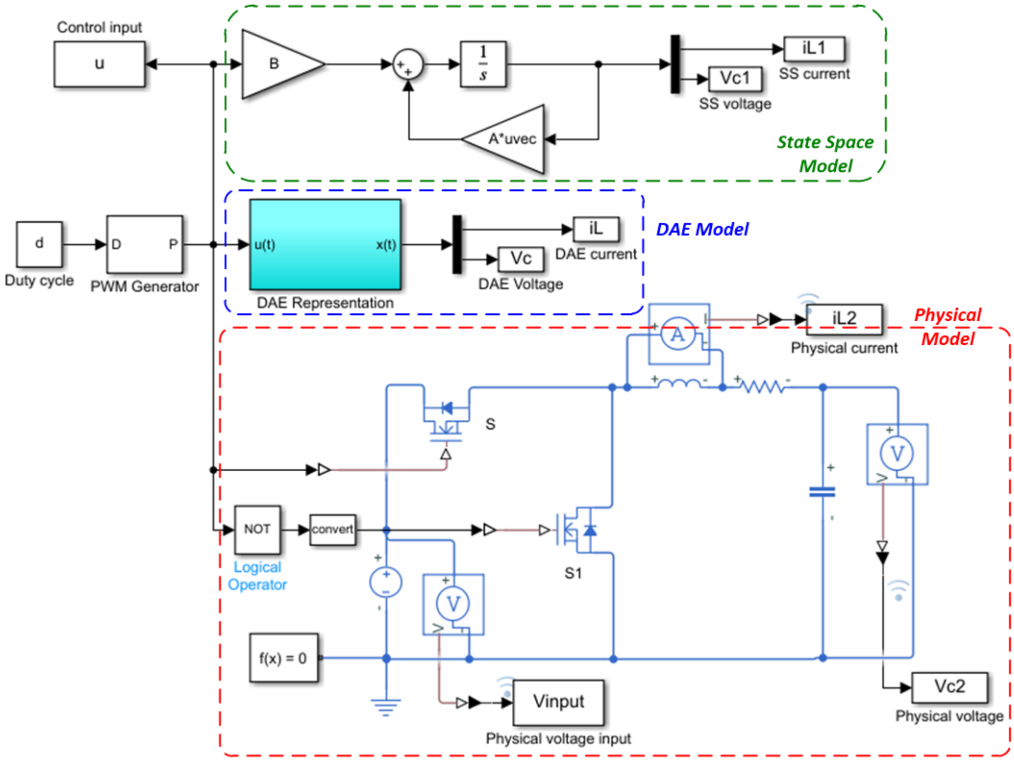

Figure 2. MATLAB SIMULINK/SIMSCAPE model used in simulation

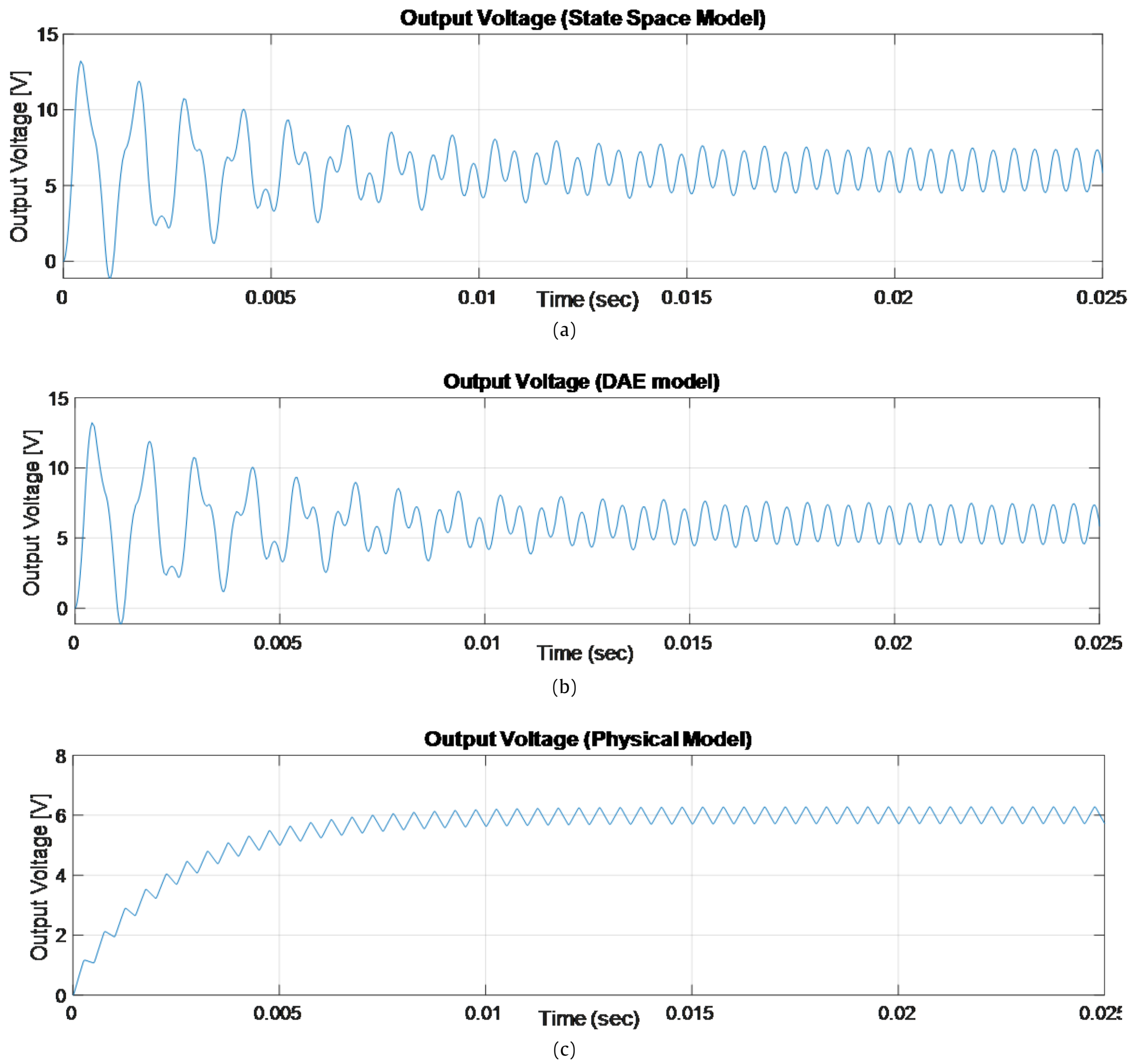

Figure 3. Output comparison of the state: (a) space; (b) DAE; and (c) physical models

IV. Conclusion

This paper has presented a convex optimization approach for the analysis of an equivalent SHS representation of DC-DC buck converter model. The proposed method first transforms the hybrid dynamics of the buck converter into an equivalent polynomial differential algebraic equation (DAE) model. The method then formulates an SOS programming algorithm for searching a Lyapunov functions which satisfy a dissipation inequality condition on the obtained DAE model that is sufficiently required to guarantee the asymptotic stability of the equilibrium point of the SHS model. Numerical simulation results show that the proposed method can prove the stability of the system in a relatively shorter computational time without relying on exhaustive simulations of the systems' dynamics. Future works will extend the proposed approach to synthesize stabilizing controller for the SHS method. Other possible directions include the implementation of other polynomial optimization approaches such as the method of moments for characterizing the stability property of SHS models as well as analyzing more complex and nonlinear hybrid model of switched hybrid power converters.

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Declarations

Author contribution

T.A. Tamba: Writing - Original Draft, Review & Editing, Conceptualization, Formal analysis, Investigation, Visualization, Supervision. J. Chandra: Validation, Data Curation. B. Hu: Resources, Software.

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Competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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