



# Study on the characteristics of pipe buckling strength under pure bending and external stress using nonlinear finite element analysis

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## Abstract

Buckling and collapse are important failure modes for laying and operating conditions in a subsea position. The pipe will be subjected to various kinds of loads, i.e., bending moment, external pressure, and tension. Nonlinear finite element analysis was used to analyze the buckling strength of the pipe under pure bending and external pressure. The buckling of elastic and elasto-plastic materials was also studied in this work. The buckling strength due to external pressure had decreased and become constant on the long pipe when the length-to-diameter ratio ( $L/D$ ) was increased. The non-dimensional parameter ( $\beta$ ), which is proportionate to  $(D/t)(\sigma_y/E)$ , is used to study the yielding influence on the buckling strength of pipe under combined bending and external pressure loading. The interaction curves of the buckling strength of pipe were obtained, with various the diameter-to-thickness ratio ( $D/t$ ) under combination loads of external pressure and bending moment. For straight pipes  $L/D = 2.5$  to 40,  $D = 1000$  to 4000 mm, and  $D/t = 50$  to 200 were set. The curved pipes  $D/t = 200$ ,  $L/D = 2.5$  to 30 have been investigated by changing the radius of curvature-to-diameter ratio ( $R/D$ ) from 50 to  $\infty$ , for each one. With decreasing  $R/D$ , the buckling strength under external pressure decreases slightly. This is in contrast to the bending of a curved pipe. When the value of  $R/D$  was decreased, the flexibility of the pipe was increased. However, the buckling strength of the pipe during bending was decreased due to the oval deformation at the cross-section.

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Keywords: buckling strength; elastic buckling; elasto-plastic buckling; bending moment; external pressure.

## I. Introduction

Offshore pipelines will be subjected to a variety of loads, including bending moment, tension, internal pressure, and external pressure. The maximum permitted bending moment was calculated using a set of equations that included proposed safety factors for various safety levels [1]. The safety factor technique used ensures that the intended safety values are maintained consistently across all load combinations. A theoretical technique for predicting the moment rotation response of circular hollow steel tubes with various  $D/t$  ratios under pure bending has been proposed [2]. In deriving of the deformation energy, extensional

deformation and rigid plastic material behavior were considered.

Previous study [3] shown that for a straight pipe with an  $L/D$  ranging from 5 to 20, and a  $D/t$  ranging from 50 to 200, the critical bending moment in a linear calculation is described by equation (1):

$$M_{cr} = 0,666\pi Ert^2 \quad (1)$$

where  $E$ ,  $r$ ,  $t$ , and  $M_{cr}$  are the Young's modulus (MPa), radius of cylinder (mm), wall thickness (mm), and the critical bending moment (Nmm), respectively.

Buckling is the sudden change in the shape of a structural component under load. It happens when a force presses on a slender structure and makes it collapse. Under bending, the buckling strength of straight and curved pipes [4] was examined. Equation (2) shows the maximum moment,  $M_{Max}$

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(Nmm), for a long perfect pipe with oval deformation:

$$M_{Max} = 0,52M_{cr} \quad (2)$$

In deep underwater pipes, the interplay of propagation buckling and lateral buckling has been investigated [5]. Propagation buckling is a local mode that can swiftly spread and destroy a lengthy pipeline segment in deep water. In contrast, lateral buckling is a probable global buckling mode in long pipelines. A numerical analysis was carried out to simulate buckling contact in deep undersea pipelines. The buckling of pipes under external and internal pressures, both elastic and non-elastic, has been studied [6]. The buckling pressure of pipes due to external/internal pressures is evaluated using an analytical model. The results show that the initial ovality significantly influences bifurcation pressure.

The buckling behavior of steel cylindrical shells (pipes, tubes, and pressure vessels) actuated by combining axial compression and external lateral pressure, which was studied using the generalized beam theory (GBT) [7]. There are comparisons made between experimental and numerical results. The empirical formula for buckling propagation pressure of offshore pipelines with various diameter-to-thickness ratios and different strain hardening modulus and yield stress is proposed [8] based on experimental and comprehensive numerical data. Finite element analysis (FEA) is used to evaluate the response of a pipe-in-pipe (PIP) system to the combined effect of external pressure (on the outer pipe) and bending moment. The FE analysis is carried out on a PIP system chosen from various offshore pipeline applications. It is demonstrated that the external pressure-induced reduction in bending moment capacity of PIP systems is greater than that of the identical single outer system without an inner pipe [9]. A parametric analysis was done using the verified FE model, and two primary buckling propagation modes were detected a pipe with thin and moderately thin carrier pipes [10]. The nonlinear FE approach investigates the local buckling failure of the damaged subsea pipeline under combined stresses. The simulated results reveal that external pressure and axial force can significantly affect the pipeline's buckling behavior and bending capacity [11].

The case of pure bending moment is investigated, and it is discovered that increasing the initial denting displacement and the diameter-to-thickness ratio reduces the critical moment. The external pressure is then applied, and it is determined that when the pre-applied external pressure and the initial denting displacement grow, the non-dimensionalized critical bending moment decreases. The collapse pressure would be reduced to some extent if the bending moment was supplied to the dented pipe before the external pressure [12].

The problem of pipeline collapse under point load, longitudinal bending, and external pressure was explored [13] using the rational model methodology and comparing anticipated results to previously published full-scale experimental data on the subject. The rational model methodology is

recommended for design codes because of its rational derivation and high prediction capabilities. The phenomena identified in the literature and industry standards as a determinant in the evaluation of flexible pipes collapsing under combined bending and external pressure were investigated [14]. The final collapse pressure is calculated by combining dimensions fluctuations and ovalization due to bending.

The results of the comparison of numerical and analytical predictions suggest that analytical methodologies can be used to predict the curve collapse of flexible pipes. The buckling behavior of long cylindrical steel shells under simultaneous bending and uniform peripheral pressure was examined in an experimental investigation [15]. In terms of buckling load and modes of deformation, the theoretical consequences predicted by nonlinear FEA accord reasonably well with the experimental data.

This paper studied nonlinear FE software used to calculate the buckling and collapse strength of straight and curved pipes under bending and external pressure, taking into account the effect of a cross-sectional oval deformation.

## II. Materials and Methods

### A. Calculation parameters

For straight pipe, the length-to-diameter ratio ( $L/D$ ) and the diameter-to-thickness ratio ( $D/t$ ) are employed as calculating factors. The diameter changes from 1000 to 4000 mm, while the  $L/D$  ranges from 2.5 to 40.  $D/t$  might range from 50 to 200. Where  $D$  is the pipe diameter in millimeters,  $t$  is the wall thickness in millimeters, and  $L$  is the pipe length in millimeters (mm).

The pipe diameter is initially set to 4000 mm, and the thickness is set to 20 mm in the computation of a curved pipe. By varying the pipe length,  $L/D$  can range from 2.5 to 30. ( $R/D$ ) fluctuates between 50 and 200. In a curved pipe,  $R$  is the radius of curvature (mm).

### B. Model for computation and program for calculation

In FEA, full-length models of straight and curved pipes are used. Calculations of nonlinear buckling of pipe under bending and external pressure are carried out. Msc Marc was utilized to do a nonlinear buckling study that included the cross-sectional oval deformation before to buckling.

The element with four nodes in a quadrilateral (No. 75) is used. In a circumferential direction, the calculation region is divided into 36 components. In the case of a long cylinder, the element count is essentially 120, and more elements are used to keep the calculation accurate.

### C. Boundary condition and loading condition

As shown in Figure 1, the ( $X, Y, Z$ ) coordinates were used to connect the center of a circle and the points on a circle, and rigid body elements (RBE) are put at both ends of the section. The bending moment is loaded at the circle's center at both ends. The rigid

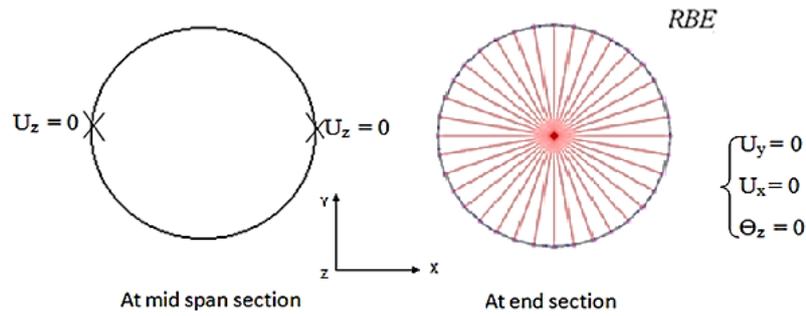


Figure 1. Mid span section &amp; RBE at both end section

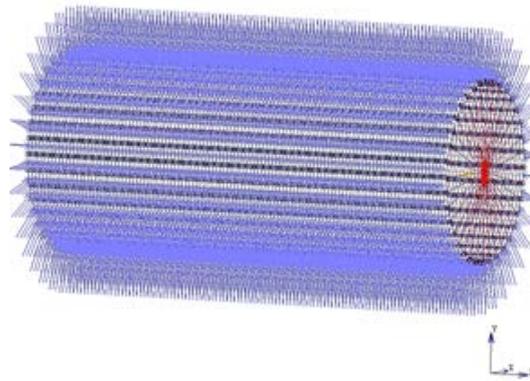
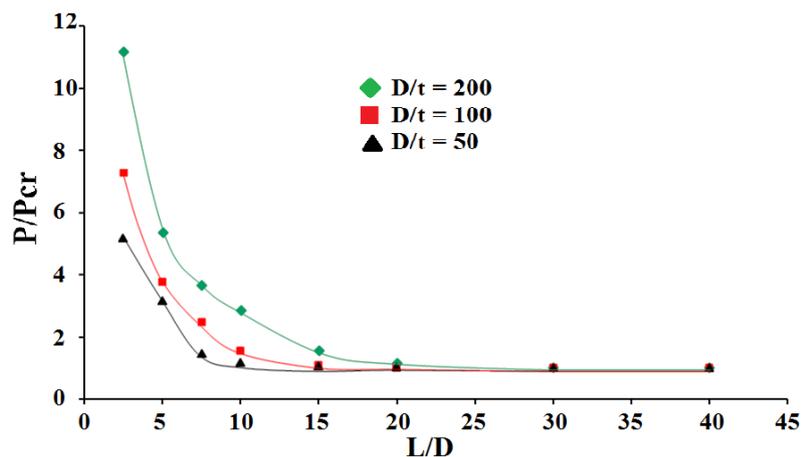


Figure 2. The external pressure is loaded uniformly at the surface of the pipe

Figure 3. Relationship between non-dimensional pressure and  $L/D$  for straight pipe ( $D/t=50-200$ ) by elastic analysis.

body elements (RBE) retain the section in-plane during rotational deformation caused by the bending moment and prevent oval deformation of both ends. Tying or RBE can be used to create a rigid link in Msc Marc for little or significant deformations. As shown in Figure 2, the external pressures are loaded uniformly at the surface of the pipe.

### III. Results and Discussions

#### A. The numerical results on nonlinear buckling strength of straight pipe under bending and external pressure

When the pipe acts elastically, Figure 3 depicts the correlations between non-dimensional pressure and pipe length. The non-dimensional pressure ( $P/P_{cr}$ ) is shown on the vertical axis, with the critical

pressure stated in equation (3). The horizontal axis is length of pipe.

$$P_{cr} = \frac{E'}{4} \left( \frac{t}{r} \right)^3 \quad (3)$$

$$E' = \frac{E}{(1-\nu^2)}$$

where  $E'$ ,  $\nu$ , and  $P_{cr}$  are the Young's modulus (MPa), poisson ratio, and the critical external pressure (MPa), respectively.

Similarly, the buckling strength of a shorter pipe under external pressure is greater than that of a long pipe. The effect of limitation at both ends of a short pipe is greater than that of long pipes. With rising  $L/D$ , the buckling strength of straight pipe subjected to external pressure decreases until it reaches a constant value at the long pipe. When  $D/t$  lowers, the critical pressure rises.

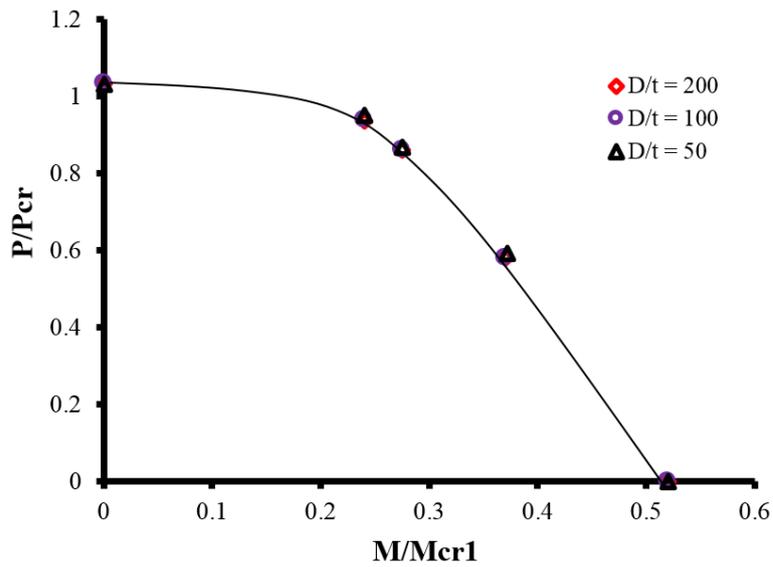


Figure 4. Moment-pressure interaction stability for straight pipe ( $L/D=30$ ) by elastic analysis

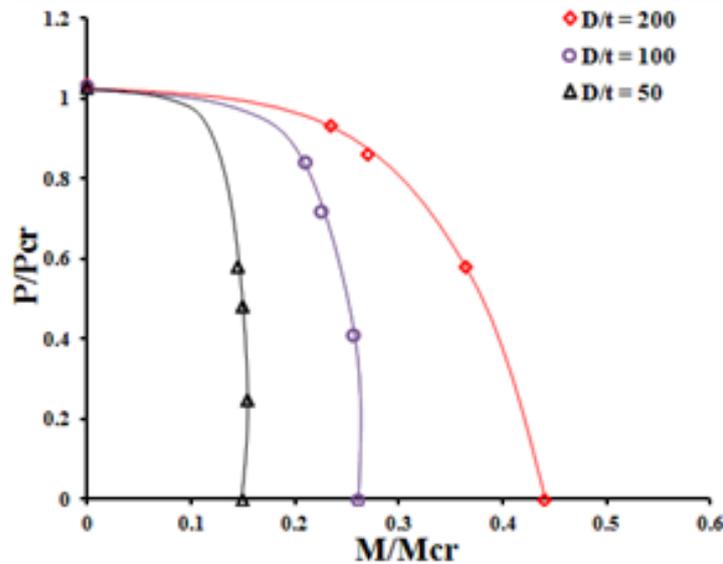


Figure 5. Moment - pressure interaction stability for straight pipe in elasto-plastic analysis

In elastic analysis, the moments - pressure interaction stability for straight pipe is shown in Figure 4. The non-dimensional pressure ( $P/P_{cr}$ ) is shown on the vertical axis, while the non-dimensional moment ( $M/M_{cr1}$ ) is shown on the horizontal axis. As indicated in equation (1),  $M_{cr1}$  is the critical bending moment of a cylinder under axial compression. For every value of  $D/t$ , the interaction curve's tendency on buckling strength under combined bending and external pressure loading was the same.

When the yield strength of the material is 621 MPa, the moments - pressure interaction stability for straight pipe in elasto-plastic analysis is presented in Figure 5. The numerical results on buckling strength of straight pipe under external pressure in elasto-plastic analysis occurred in the elastic zone. Pipe with a big  $D/t$  value is more elastic than pipe with a small  $D/t$  value under combined bending and external pressure loading.

The non-dimensional parameter ( $\beta$ ) as stated in equation (4) is used to examine the yielding

influence on the buckling strength of pipe under pure bending and external pressure.

$$\beta = \left(\frac{D}{t}\right)\left(\frac{\sigma_y}{E}\right) \tag{4}$$

where  $\beta$  and  $\sigma_y$  are non-dimensional parameter, and the yield stress (MPa), respectively. This parameter is determined by the linear buckling moment to initial yielding moment ratio. Changes in yield stress and diameter are used to test the elasto-plastic buckling strength.

In an elasto-plastic analysis, the moments-pressure interaction stability for straight pipe is illustrated in Figure 6. The lines with the hollow diamond, hollow circle, and hollow triangle markers represent the numerical results of  $D/t=200, 100, 50$ , and  $y=621$ , where  $\beta$  equals 0.6, 0.3, and 0.15 respectively. Moreover, the solid diamond marker are the numerical results by  $D/t=100$  and  $\sigma_y=1260$ ,  $\beta$  equals 0.6. And then, for the solid circle and solid triangle maker are the numerical results by  $D/t=200$  and  $\sigma_y=315$  MPa and 157.5 MPa, where  $\beta$  equals 0.3

and 0.15. The pipe buckles elastically when the value of is large, and the pipe buckles elasto-plastically when the value of is small.

B. The numerical studies on the nonlinear buckling strength of curved pipe under bending and external pressure

According to elastic analysis, the relationship between non-dimensional pressure and  $L/D$  for curved pipe ( $D/t = 200$ ) and  $R/D$  fluctuates between 50 and 200, as illustrated in Figure 7. Similarly, as the  $L/D$  increases, the buckling strength of a curved pipe decreases until it reaches a constant value on the long pipe. On a short pipe, the effects of limitation at both ends are considerable for a curved pipe under external pressure. However, for a curved pipe with the same  $D/t$  and a different  $R/D$ , the buckling strength values were nearly identical for each same  $L/D$  value, or the differences were not significant. This is in contrast to the bending of a curved pipe.

When the value of  $R/D$  decreases, the flexibility of the pipe increases, but the buckling strength of the pipe during bending decreases due to the oval deformation at the cross-section. In elastic analysis,

the moments - pressure interaction stability for curved pipe is shown in Figure 8. The non-dimensional pressure ( $P/P_{cr}$ ) is shown on the vertical axis, while the non-dimensional moment ( $M/M_{cr1}$ ) is shown on the horizontal axis. With a lower  $R/D$  number, the buckling strength decreases.

When the yield strength of the material is 621 MPa, as represented in Figure 9, moments - pressure interaction stability for curved pipe in elasto-plastic analysis. The pipe with a large  $R/D$  value is more elastic than the pipe with a small  $R/D$  value. When the value of  $R/D$  is large for curved pipe under combined loading of bending and external pressure, the pipe is undoubtedly more elastically than when the value of  $R/D$  is small. Under external pressure, the buckling strength of a straight pipe is about the same for each  $D/t$ . The buckling strength under external pressure is slightly reduced with lowering  $R/D$  for a curved pipe, as shown in Figure 7, with the same value of  $D/t$  and difference  $R/D$ . However, the disparity isn't significant. This is in contrast to the bending of a curved pipe. When the value of  $R/D$  decreases, flexibility increases, oval deformation at mid-span increases, and buckling strength decreases.

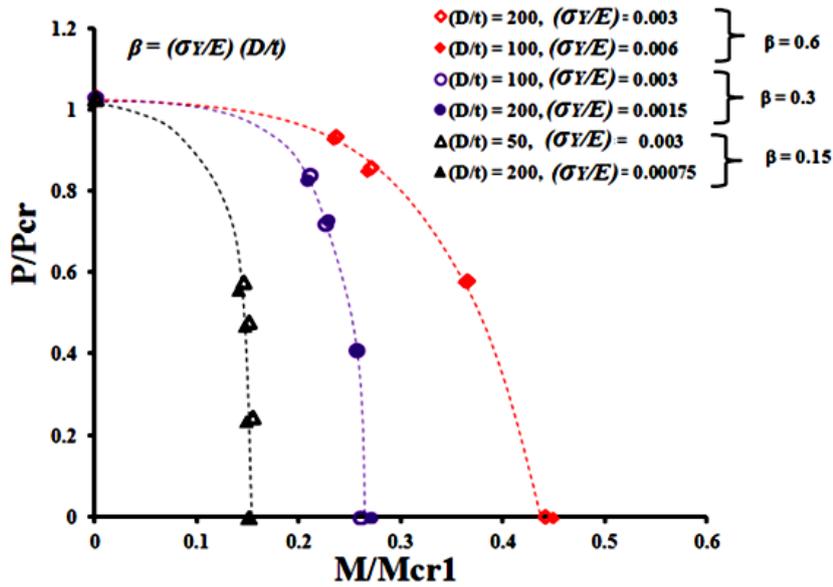


Figure 6. Moment-pressure interaction stability for straight pipe in elasto-plastic analysis

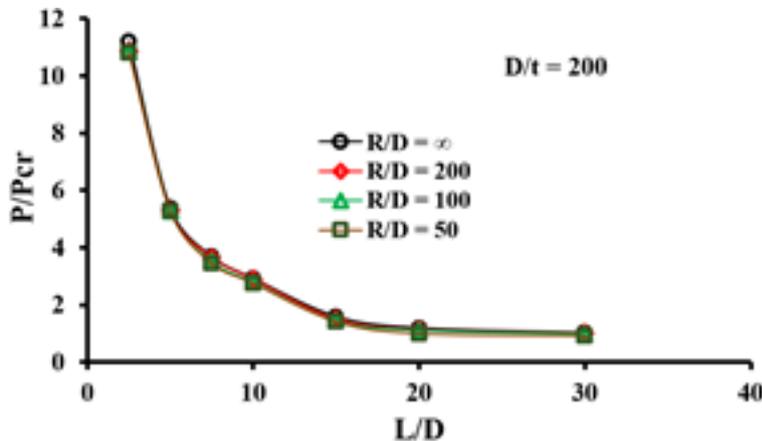


Figure 7. Relationship between non-dimensional pressure and  $L/D$  for curved pipe ( $D/t=200$ ) by elastic analysis.

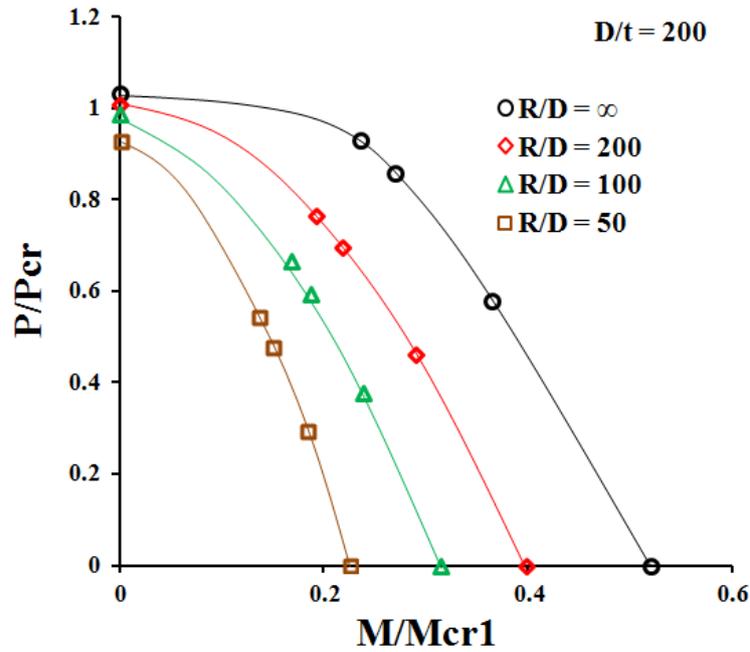


Figure 8. Moment-pressure interaction stability for curved pipe in elastic analysis

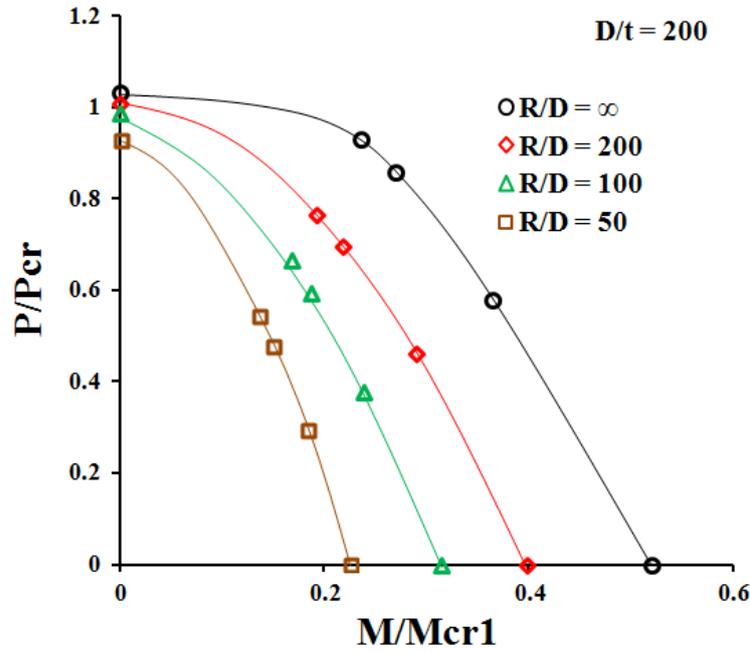


Figure 9. Moment-pressure interaction stability for curved pipe in elasto-plastic analysis

#### IV. Conclusion

The study of nonlinear FE software is used to calculate the buckling and collapse strength of straight and curved pipes under combined loads, such as bending and external pressure. The numerical computation clarifies the following. In elastic analysis, when  $L/D$  increases, the buckling strength decreases owing to external pressure and becomes constant on the long pipe. In the shorter pipe, the effect of limitation at both ends is greater than in the longer pipe. For straight pipe under external pressure, the buckling strength occurs on the elastic region. A pipe with a high  $D/t$  value is more elastic than a small  $D/t$  value under combined

bending and external pressure loading. For every value of  $D/t$ , the tendency of the interaction curve on buckling strength for straight pipe under combined bending and external pressure was the same in elastic analysis. The non-dimensional parameter ( $\beta$ ) is used to examine the yielding influence on pipe buckling strength. When a curved pipe has the same  $D/t$  value but a different  $R/D$  (where  $R/D$  ranges from 50 to  $\infty$ ), the buckling strength under external pressure decreases slightly as  $R/D$  decreases. However, the disparity isn't significant. This is in contrast to the bending of a curved pipe. When the value of  $R/D$  decreases, the flexibility of the pipe increases, but the buckling strength of the pipe during bending decreases due to the oval deformation at the cross-section.

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## Declarations

### Author contribution

Hartono Yudo and Tri Admono contributed as the main contributors of this paper. All authors read and approved the final paper.

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### Conflict of interest

The authors declare no known conflict of financial interest or personal relationships that could have appeared to influence the work reported in this paper.

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