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Design and application of models reference adaptive control (MRAC) on ball and beam

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Abstract

This paper presents the implementation of an adaptive control approach to the ball and beam system (BBS). The dynamics of a BBS are non-linear, and in the implementation, the uncertainty of the system's parameters may occur. In this research, the linear state-feedback model reference adaptive control (MRAC) is used to synchronize the states of the BBS with the states of the given reference model. This research investigates the performance of the MRAC method for a linear system that is applied to a non-linear system or BBS. In order to get a faster states convergence response, we define the initial condition of the feedback gains. In addition, the feedback gains are limited to get less oscillation response. The results show the error convergence is improved for the different sets of the sinusoidal reference signal for the MRAC with modified feedback gains. The ball position convergence improvement of MRAC with modified feedback gains for sinusoidal reference with an amplitude of 0.25, 0.5, and 0.75 are 35.1 %, 36 %, and 52.4 %, respectively.

Keywords: model reference adaptive control; modified feedback gains; ball and beam system.

I. Introduction

In designing the control of a system, one can use the simulation and experiment to see the effectiveness of the proposed method. However, this approach may cost a lot due to the miscalculation in the control system design. Hardware in-the-loop (HIL) is a simulation technique performed by combining hardware and software in the process [1]. Implementing HIL will facilitate the testing process and reduce the level of errors or failures that occur as well as the costs required in the design of the control system [2]. In this research, we use a real plant BBS and the controller in MATLAB, which is the opposite of the HIL scheme. Using this approach, one can directly implement the proposed control method that is designed in MATLAB to a real plant.

Adaptive control is an advanced control method with parameter adjustments that can regulate the states or output of the uncertain system to track a certain value [3]. In adaptive control, the unknown system is expected to converge its states or output to reference model states or output. The model reference adaptive control (MRAC) makes the unknown system dynamics similar to the reference model dynamics [4][5]. The MRAC can be categorized into two types based on how its estimates the unknown parameters, indirect and direct MRAC [6][7]. The author uses the state-feedback direct MRAC control method with modified feedback gains to get fast adaptation. Recent research on fast adaptation in adaptive control can be found in [8].

The BBS is one of the most widely used examples of control systems application in control engineering. The primary purpose of BBS is to track the ball to the commanded position by designing a particular

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control system [9]. This research uses the direct method MRAC for a linear system that is implemented to BBS, a non-linear system. In other studies, various adaptive methods for BBS systems have been carried out. The control for the ball and beam system that combines the conventional dynamic surface control and the adaptive fuzzy scheme is proposed for the equilibrium balance of the ball [10]. There are also designing a model reference adaptive control system using the MIT rule to control a ball and beam system so the plant could track the reference model [11]. The comparison between the integer and the fractional controller for BBS has been discussed and tested in [12]. It is known that a mechatronic system has limited control input due to actuator limitations. The parameter projection algorithm is used to solve the control saturation that may lead to undesirable results in adaptive controllers [13]. Compared with most adaptive control of BBS literature, this work shows the effectiveness of direct state-feedback MRAC both in simulation and in an experiment as a real-time controller of the BBS. In addition, we proposed predefined feedback gains to have a faster convergence rate and feedback gains saturation to get a less oscillated response shown by a small state error value.

II. Materials and Methods

A. Ball and beam system

The BBS objective is to keep the ball's position in the desired location by connecting the beam to a servo motor. The ball's position is determined from the edge of the beam. The BBS configuration can be seen in Figure 1.

The motion of the ball can be found by using Newton's law that satisfies the following equation (1)

$$mg\ \sin\alpha - F_f = m\ddot{r} \tag{1}$$

where $F_f = \frac{j_b}{R^2}\ddot{r}$ is the frictional force, j_b is the moment of inertia of the ball, and α is the deflection of the beam. Here we are assuming that α is very small. Thus we have linearized BBS motion in equation (2)

$$mg\ \alpha - \frac{j_b}{R^2}\ddot{r} = m\ddot{r} \tag{2}$$

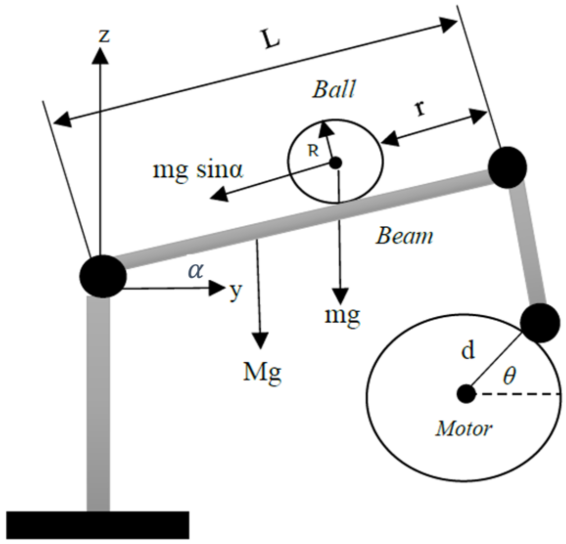

Figure 1. Ball and beam dynamics, where m is ball mass, M is rod mass, d is offset arm length, g is gravitational acceleration, R is ball radius, r is ball position, θ is angle of servo, and L is length beam

It is known that $\alpha = \frac{d}{L}\theta$, so that we have the following BBS dynamics equation (3)

$$\left(\frac{j_b}{R^2} + m\right)\ddot{r} = mg\frac{d}{L}\,\theta \tag{3}$$

Using the differential equation in equation (3), the dynamics of the second-order BBS modeling system in the state-space form can be defined as equation (4)

$$\begin{bmatrix}\dot{r}\\ \ddot{r}\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}r\\ \dot{r}\end{bmatrix} + \begin{bmatrix}0\\ \frac{mgd}{L\left(\frac{J}{R^2}+m\right)}\end{bmatrix}\theta \tag{4}$$

where $\dot{r}$ is velocity of the ball and $\ddot{r}$ is acceleration of the ball.

B. Model reference adaptive control (MRAC)

The model reference adaptive control (MRAC) is one of the adaptive control methods which aims to solve control problems with limited parameters to compensate for unknown system parameters by adapting the characteristics of the stable reference model. Thus the system has the same characteristics similar to the reference model. In this study, direct MRAC was used [14]. Figure 2 shows the structure of the direct MRAC.

In direct state-feedback MRAC, the following equations are used:

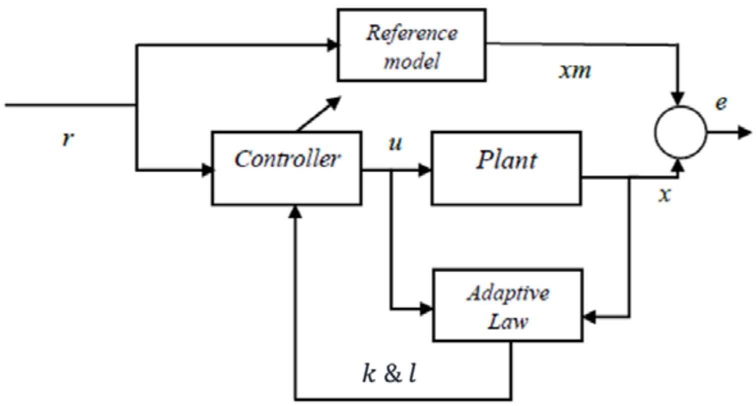

Figure 2. Direct state-feedback MRAC

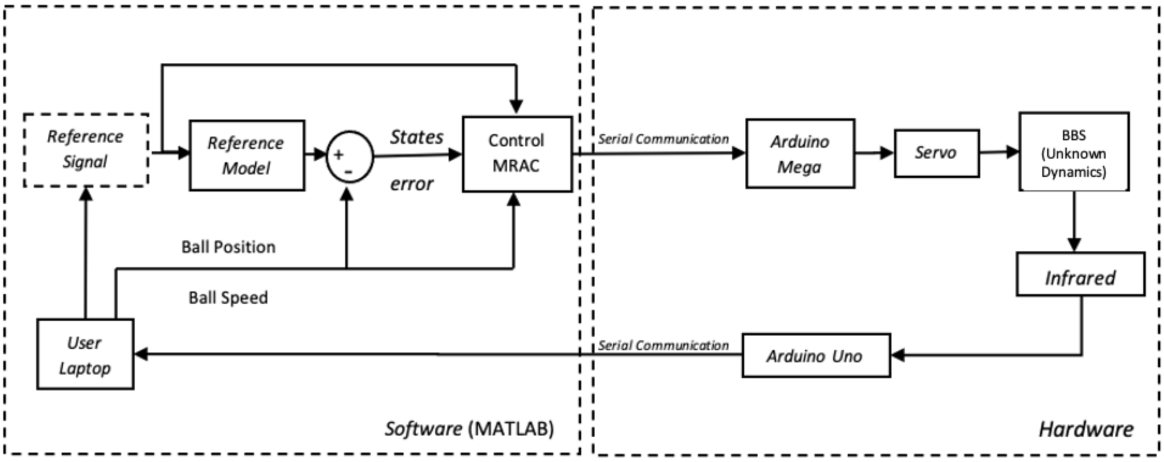

Figure 3. Block diagram system

Unknown system using equation (5)

$$\dot{x} = Ax + Bu \tag{5}$$

Reference model using equation (6)

$$\dot{x}_m = A_m x_m + B_m r \tag{6}$$

Lyapunov equation using equation (7)

$$A_m' P + P A_m = -Q, \quad Q > 0 \tag{7}$$

Adaptive Laws using equation (8) and (9)

$$\dot{\hat{k}} = \gamma_1 B'_m P e(x - x_m) x' \operatorname{sgn}(l^*) \tag{8}$$

$$\dot{\hat{l}} = -\gamma_2 B'_m P e(x - x_m) r \operatorname{sgn}(l^*) \tag{9}$$

Control Law using equation (10)

$$u = -kx + lr \tag{10}$$

where A is state matrix of unknown system, A_m is state matrix of model reference, B is input matrix of unknown system, B_m is input matrix of model reference, r is input reference, u is control law, x is system's states, x_m is reference model's states, Q & P is matrix positive definite, e is error in state feedback, γ_1 & γ_2 is adaptive gains, and k & l is feedback gains.

C. System setup

Unlike the standard hardware in-the-loop (HIL) scheme that uses a real controller to control the virtual system, we proposed the opposite of the HIL scheme. In this research, the controller is MATLAB, and the system/BBS is a real plant. Figure 3 shows the detailed system block diagram proposed in this research.

The following is an explanation of the block diagram in Figure 3:

- The initial condition is given by the user as an initial state of the BBS in SIMULINK MATLAB.
- The initial information is processed in SIMULINK MATLAB, in which MRAC control is designed.
- The resulting control signal is sent to the ball and beam system via Arduino Mega. Then the servo motor will move the beam according to the control signal command.
- The states response from the BBS is measured using an infrared sensor and sent back to MATLAB via Arduino Uno. The results are the states of the BBS, the position and the speed of the ball.
- The ball moves on the beam according to the given control and adapts the reference model that was designed previously.
- This process will continue until the SIMULINK running time finish.

D. System flowchart

The system starts by initializing its parameters and adjusting the beam's position in its equilibrium state. After the setup has been set, we run the program that triggers the infrared sensor to detect the ball's position. The measured states are used as the inputs for the MRAC besides the reference signal. The adaptive laws (8) and (9) will adaptively calculate the feedback gain. The output of the MRAC or the control law will be sent to the servo motor. Figure 4 shows the diagram of the entire BBS system.

E. State feedback direct MRAC setup

The BBS parameters are defined in MATLAB to facilitate the simulation of the BBS. The state-space modeling of BBS is designed in SIMULINK, where the BBS parameters are defined as follows:

- m = 0.148 kg
- g = 9.8 m/s^2
- L = 0.35 m

Then the state-space form of the BBS system in equation (4) can be rewritten in equation (11)

$$\begin{bmatrix}\dot{r}\\ \ddot{r}\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}r\\ \dot{r}\end{bmatrix} + \begin{bmatrix}0\\ 0.75\end{bmatrix}\theta \tag{11}$$

and the stable model reference dynamics in the state-space form is defined as equation (12)

$$\dot{x}_m = \begin{bmatrix}0 & 1\\ -0.1 & -0.2\end{bmatrix}\begin{bmatrix}x_{1m}\\ x_{2m}\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix} r \tag{12}$$

Equation (12) is the reference model dynamics where the poles are on the left half plane and located near the origin. We used the adaptive law in equations (8) and (9), where $\gamma_1 = \begin{bmatrix}0.00025\\ 0.0025\end{bmatrix}$, $\gamma_2 = [-0.0025]$, and Q = $\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$.

III. Results and Discussions

A. State feedback direct MRAC on ball and beam system experiment

First, we define the initial ball position at 17.5 cm from the edge of the beam, the sinusoidal frequency is 0.005 rad/sec, the sinusoidal bias is 1.75, and the SIMULINK run time is 10,000 seconds. The responses based on the simulation and experiment of the proposed MRAC design for BBS can be seen in Figure 5. In the experiment (blue line), the state error between simulation and experiment is 7.65 % for an amplitude of 0.25, 13.41 % for an amplitude of 0.5, and 15.14 % for an amplitude of 0.75. It can be seen in Figure 6 that the states of the BBS can converge to the states of the model reference with small errors at different timescales marked by dotted lines. Therefore, we can take the values the feedback gains, k and l, at that time range as the initial and saturation values.

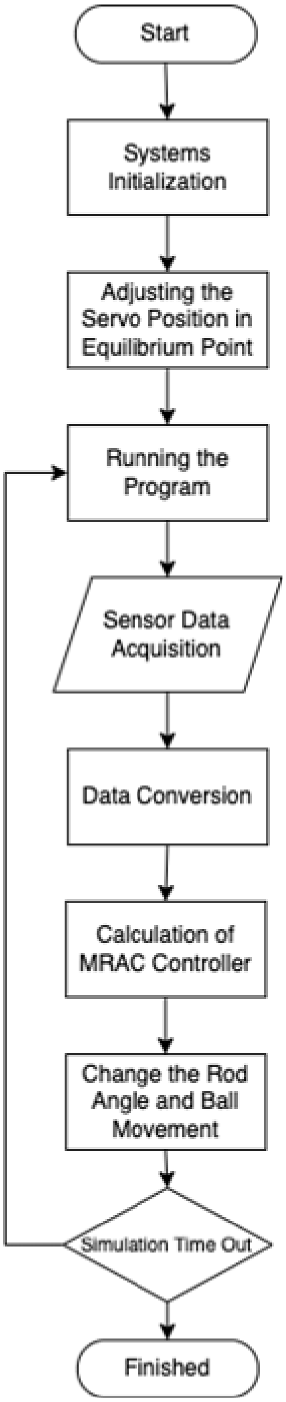

Figure 4. BBS flowchart

B. The influence of the Initial definition and saturation value on the BBS system

From the previous results, we get the information to define the initial value and the saturation value of the feedback gains k and l. For sinusoidal reference signal with an amplitude of 0.25, we have $k_1 \in [5,10]$, $k_2 \in [50,100]$, and $l \in [40,80]$. For sinusoidal reference signal with an amplitude of 0.5, we have $k_1 \in [8,10]$, $k_2 \in [35,98]$, and $l \in [40,80]$. For sinusoidal reference signal with an amplitude of 0.75, we have $k_1 \in [5,12]$, $k_2 \in [40,140]$, and $l \in [40,100]$.

The responses, simulation and experiment, of the proposed MRAC with modified feedback gains k and l can be seen in Figure 6. The yellow signal color indicates the reference signal, the blue color indicates the BBS experimental signal, and the red dotted line indicates the simulation signal. In the experiment (blue line), the states error between simulation and experiment is 4.97 % for an amplitude of 0.25, 8.57 % for an amplitude of 0.5, and 7.21 % for an amplitude of 0.75.

Table 1 shows the state error (error position of the ball) value before and after we define the initial and the saturation values of the feedback gains k and l. It can be concluded that the modified feedback gains k and l decreased the error value by 2.68 % in the case of sinusoidal with an amplitude of 0.25, 4.84 % in the case of sinusoidal with an amplitude of 0.5, and 7.93 % in the case of sinusoidal with an amplitude of 0.75. Figure 7 shows the comparison of the ball position when using the MRAC with modified feedback gains and standard MRAC.

Table 2 shows the performance improvement in terms of ball position error after the modification of the initial and the saturation values of feedback gains k and l. It can be concluded that the modified feedback gains k and l gives the biggest improvement in terms of ball position error when the sinusoidal amplitude is 0.75. The smaller amplitude gives a lower performance improvement.

Table 1.
Comparison of the difference of ball position error in the experiment

Amplitude	$\boldsymbol{k}$ and $\boldsymbol{l}$ (% *error*)	Modified $\boldsymbol{k}$ and $\boldsymbol{l}$ (% *error*)	The difference (% *error*)
0.25	7.65 %	4.97 %	2.68 %
0.5	13.41 %	8.57 %	4.84 %
0.75	15.14 %	7.21 %	7.93 %

Table 2.
Performance improvement MRAC with modified feedback gains $\boldsymbol{k}$ and $\boldsymbol{l}$ in the experiment

Amplitude	Performance improvement with modified $\boldsymbol{k}$ and $\boldsymbol{l}$ (in percentage)
0.25	35.1 %
0.5	36 %
0.75	52.4 %

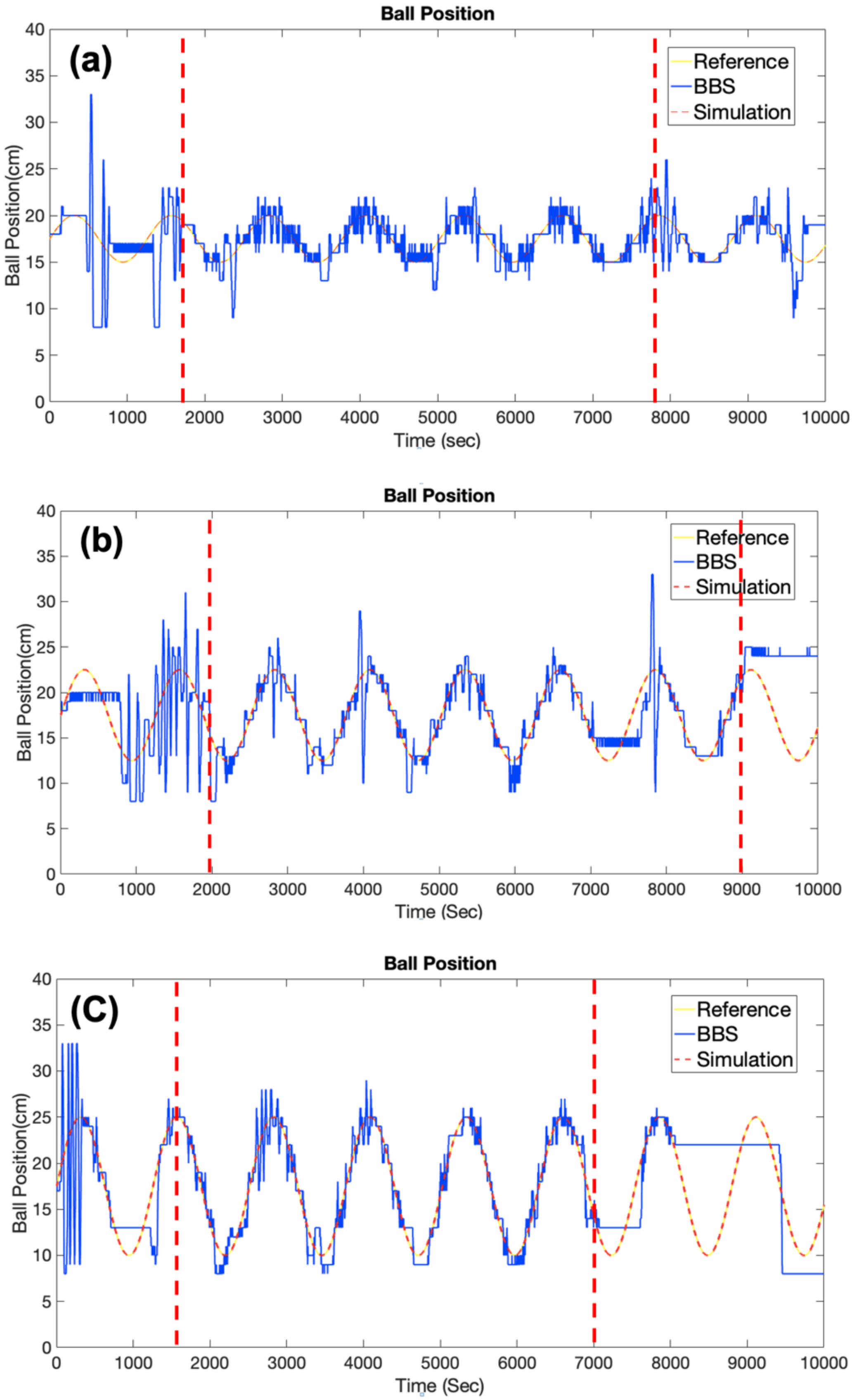

Figure 5. Comparison of simulations and experiments of BBS without initial definition and saturation values at the amplitude of (a) 0.25; (b) 0.5; and (c) 0.75

Figures 5 and 6 show the simulation responses are better than the experimental responses. This happens because the BBS model in simulation is defined as a linear system and is unaffected by any disturbances. In the experiment, the non-linear dynamics cannot be neglected, but it can be seen that the MRAC with modified feedback gains gives a better response or synchronization.

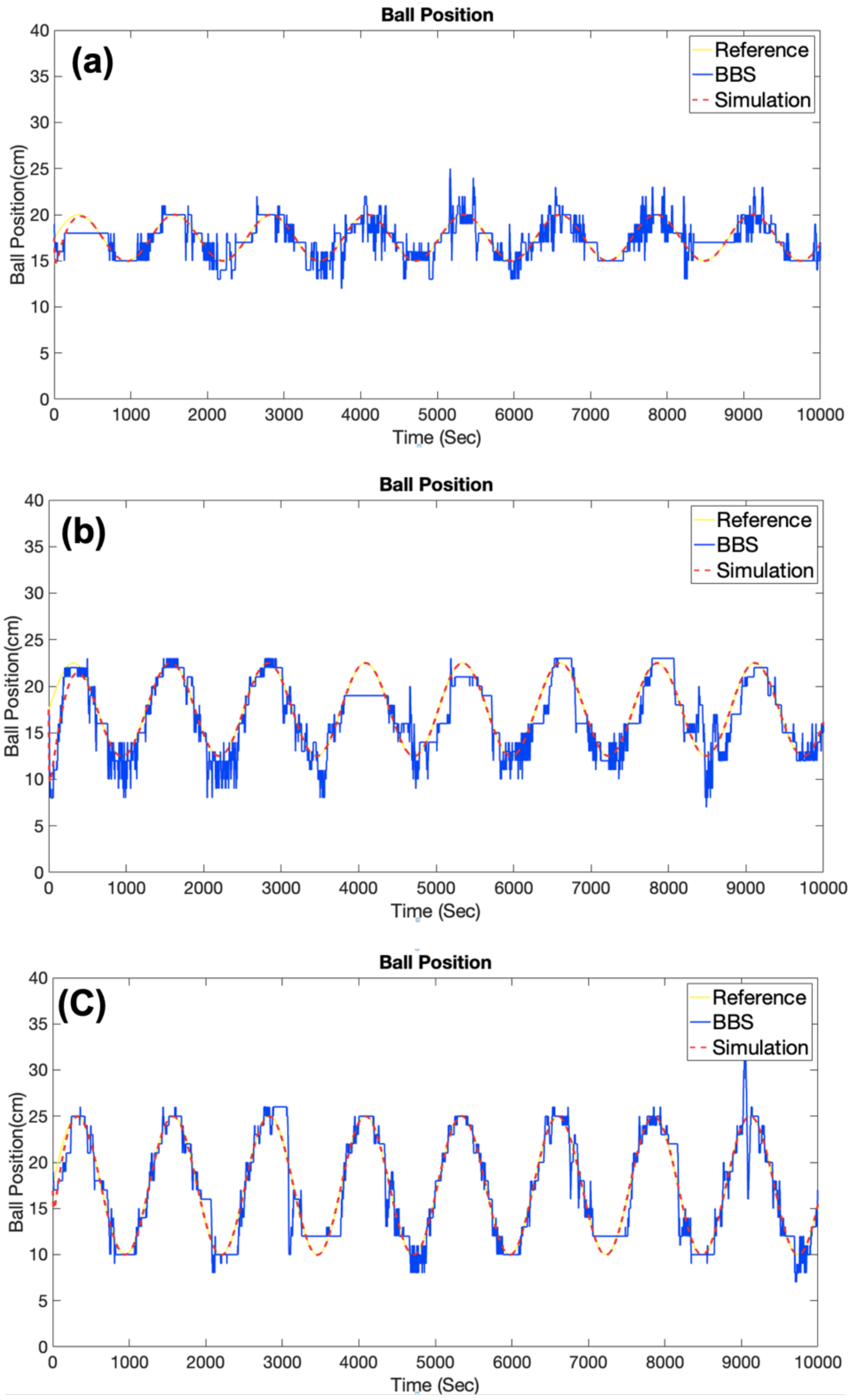

Figure 6. Comparison of simulations and experiments of BBS with initial definition and saturation values at the amplitude of (a) 0.25; (b) 0.5; and (c) 0.75

Table 3 shows the ball position error both in simulation and experiment. It is shown that the designed MRAC with modified feedback gain gives a different response error between the simulation and experiment of less than 10 %. We have different responses error between the simulation and

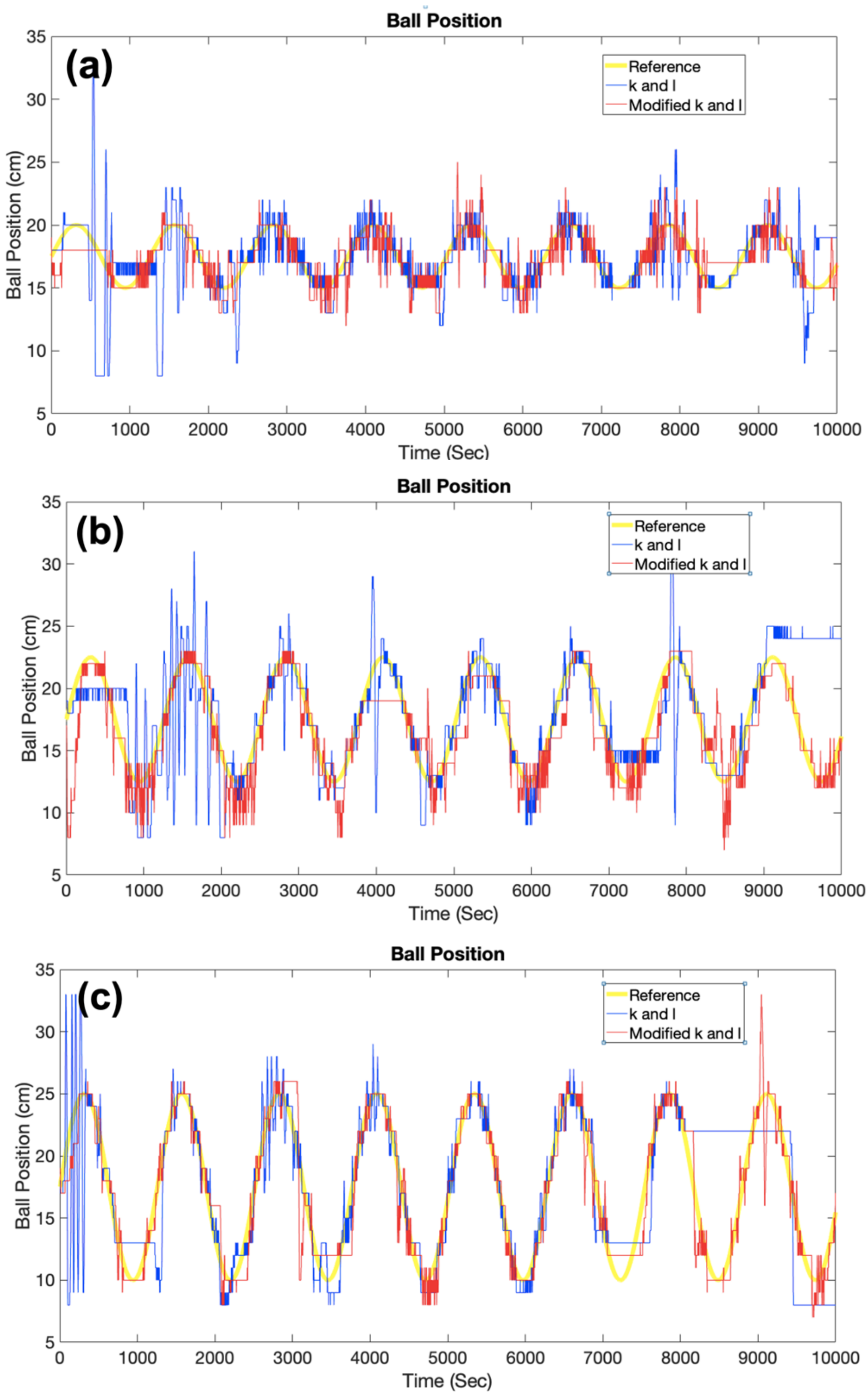

Figure 7. Comparison of the ball position when using the MRAC with modified feedback gains and standard MRAC in the experiment at the amplitude of (a) 0.25; (b) 0.5; and (c) 0.75

experiment in the case of sinusoidal with an amplitude of 0.25, 0.5, and 0.75 equal to 4.37 %, 6.37 %, and 6.56 %, respectively. Future work may include implementing adaptive control that takes into account the input saturation [15], where we know that the BBS system has limited servo actuation.

Table 3.
Comparison of the ball position error values in simulations and experiments BBS

Amplitude	Simulation (*% error*)	Experiment (*% error*)	Difference between simulation and experiment (*% error*)
0.25	0.004 %	7.65 %	7.646 %
0.25-modified k and l	0.6 %	4.97 %	4.37 %
0.5	0.0036 %	13.41 %	13.4064 %
0.5-modified k and l	2.2 %	8.57 %	6.37 %
0.75	0.004 %	15.14 %	15.136 %
0.75-modified k and l	0.65 %	7.21 %	6.56 %

IV. Conclusion

In this research, we have shown that the modified feedback gains are able to make the system performance better, which is shown by a smaller error value. This work shows the experiment result of the MRAC and we proposed the modified feedback gains k and l. The state-feedback MRAC with modified feedback gains k and l experiment resulted in a smaller ball position error with lower error percentage value of 2.68 % for the sinusoidal amplitude of 0.25, 4.84 % for the sinusoidal amplitude of 0.5, and 7.93 % for the sinusoidal amplitude of 0.75. The performance improvement with modified k and l (in percentage) is 35.1 % for the sinusoidal amplitude of 0.25, 36 % for the sinusoidal amplitude of 0.5, and 52.4 % for the sinusoidal amplitude of 0.75. Comparison of the simulation ball position errors is better in the simulation. This happens due to the simplified BBS model and the absence of any disturbances. The modification of the feedback gains gives better ball position convergence to the reference model, but a testing case is required to get the information on the initial values and saturation values of the feedback gains.

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Declaration

Author contribution

All authors contributed equally as the main contributor of this paper. All authors read and approved the final paper.

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Competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Additional information

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